

Space group determination

Some figures have been copied from slides of G. M. Sheldrick

Internet sources to consult: (and lots of others)

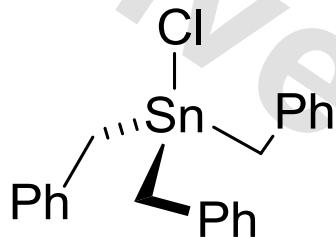
<http://img.chem.ucl.ac.uk/sgp/mainmenu.htm>

<http://subaru2.univ-lemans.fr/enseignements/physique/02/cristallo/bravais.html>

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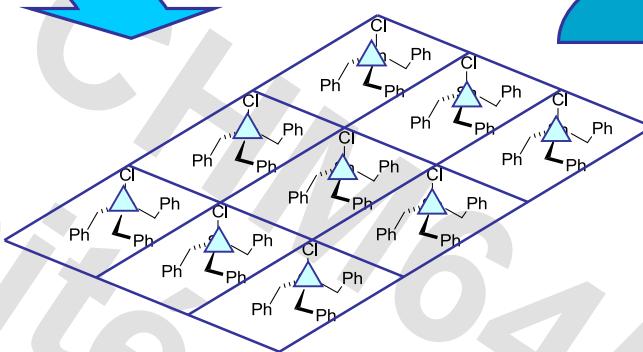
Structure determination

Crystallisation



Molecular structure:
Atomic positions

Single crystal selection

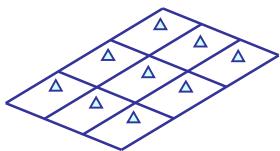


Crystal:
Macroscopic dimensions

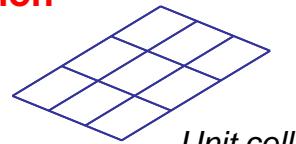
↑ Solution and
Refinement

Crystalline structure:
Unit cell and space group

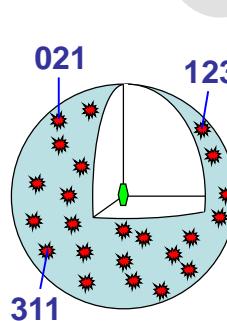
↓ Dataset
collection



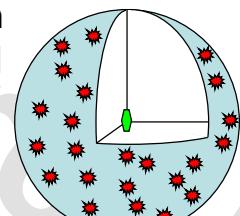
Space group
determination



Unit cell + space group:
Dimensions and symmetry of
the crystalline structure
Intensity of the reflections:
Atomic positions



Détermination
of the unit cell



H	K	L	I	σ
0	0	1	134.4	12.5
0	0	2	0.2	1.2
1	1	4	52.4	2.2

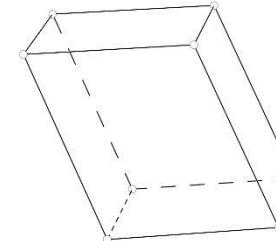
The seven crystal systems

Which possibilities exist for a unit cell ?

Triclinic

Dimension
conditions

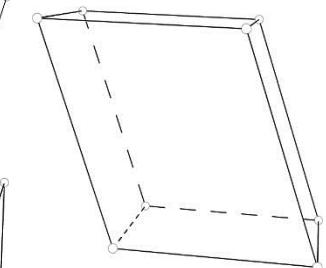
Angle
conditions



Monoclinic

-

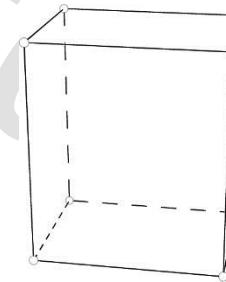
$$\alpha = \gamma = 90^\circ$$



Orthorhombic

-

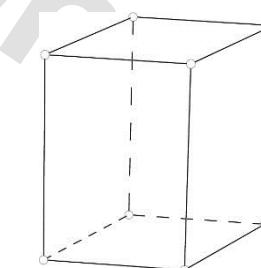
$$\alpha = \beta = \gamma = 90^\circ$$



Tetragonal

$$a = b$$

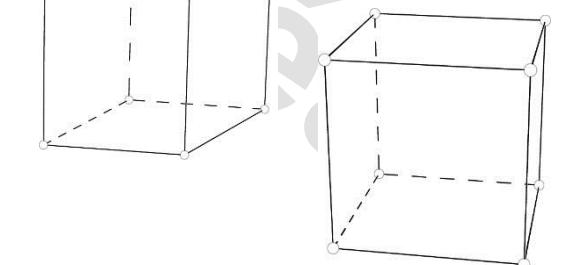
$$\alpha = \beta = \gamma = 90^\circ$$



Trigonal, hexagonal

$$a = b$$

$$\alpha = \beta = 90^\circ, \gamma = 120^\circ$$



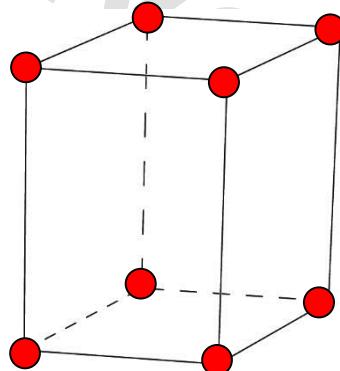
Cubic

$$a = b = c$$

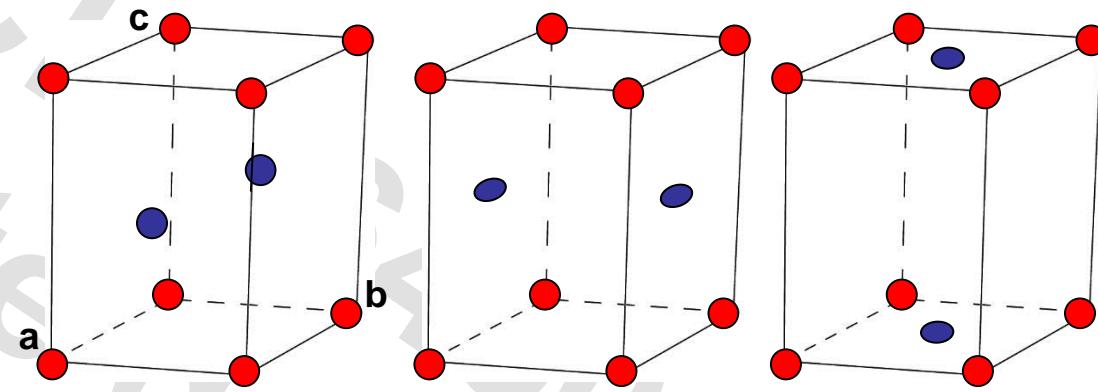
$$\alpha = \beta = \gamma = 90^\circ$$

The 14 Bravais lattices

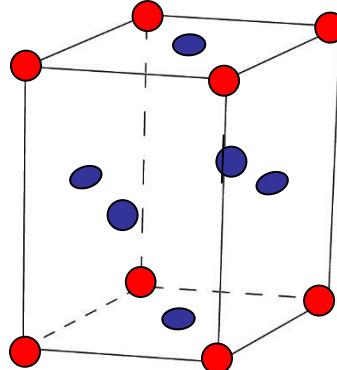
We obtain 14 Bravais lattices, when we combine the crystal systems with the centering. Centering describes that more than one “unit”/molecule is present in the unit cell (additional translational symmetry).



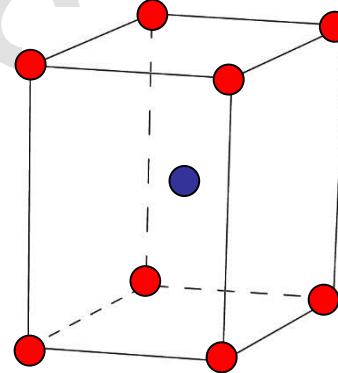
Primitive (*simple*) **P**



Face centered (*face centrée*) **A, B, C**

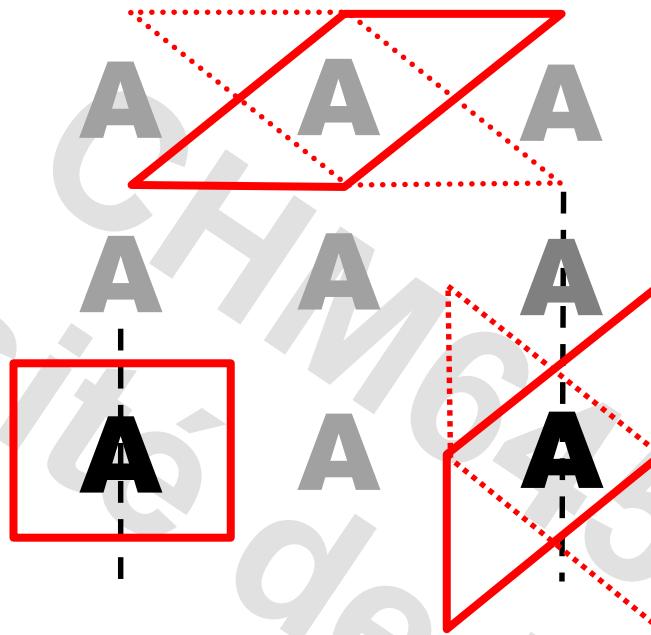


All-faces centered (*faces centrées*) **F**



Body centered (*centré*) **I**

Why do we need centered lattices ?

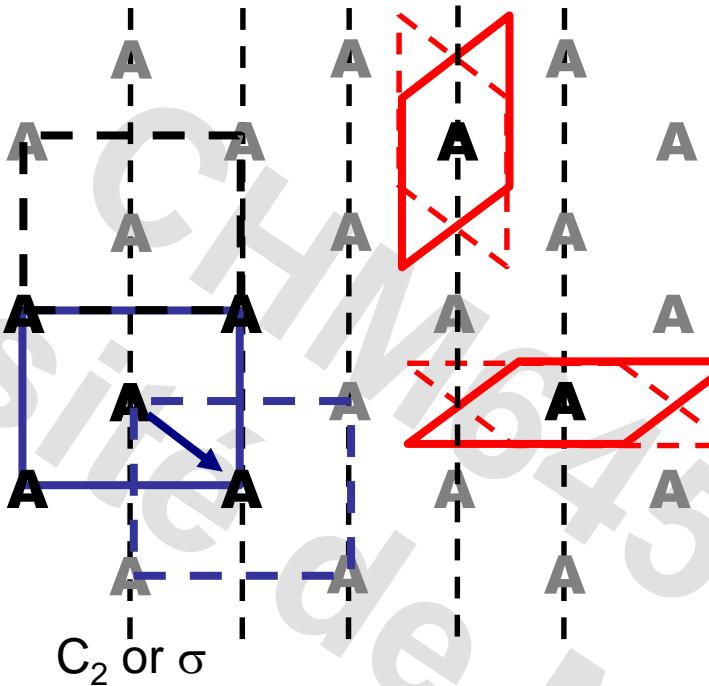


In the absence of symmetry, we can choose every possible unit cell.

If a C_2 rotation axis or a mirror plane is present, only a monoclinic unit cell (or higher) is compatible with these symmetry elements. Without that symmetry it wouldn't even be a monoclinic cell: it would be a triclinic cell with angles very, very close to 90° !

To correctly describe our structure, we have always to choose the highest possible symmetry.

Why do we need centered lattices ?

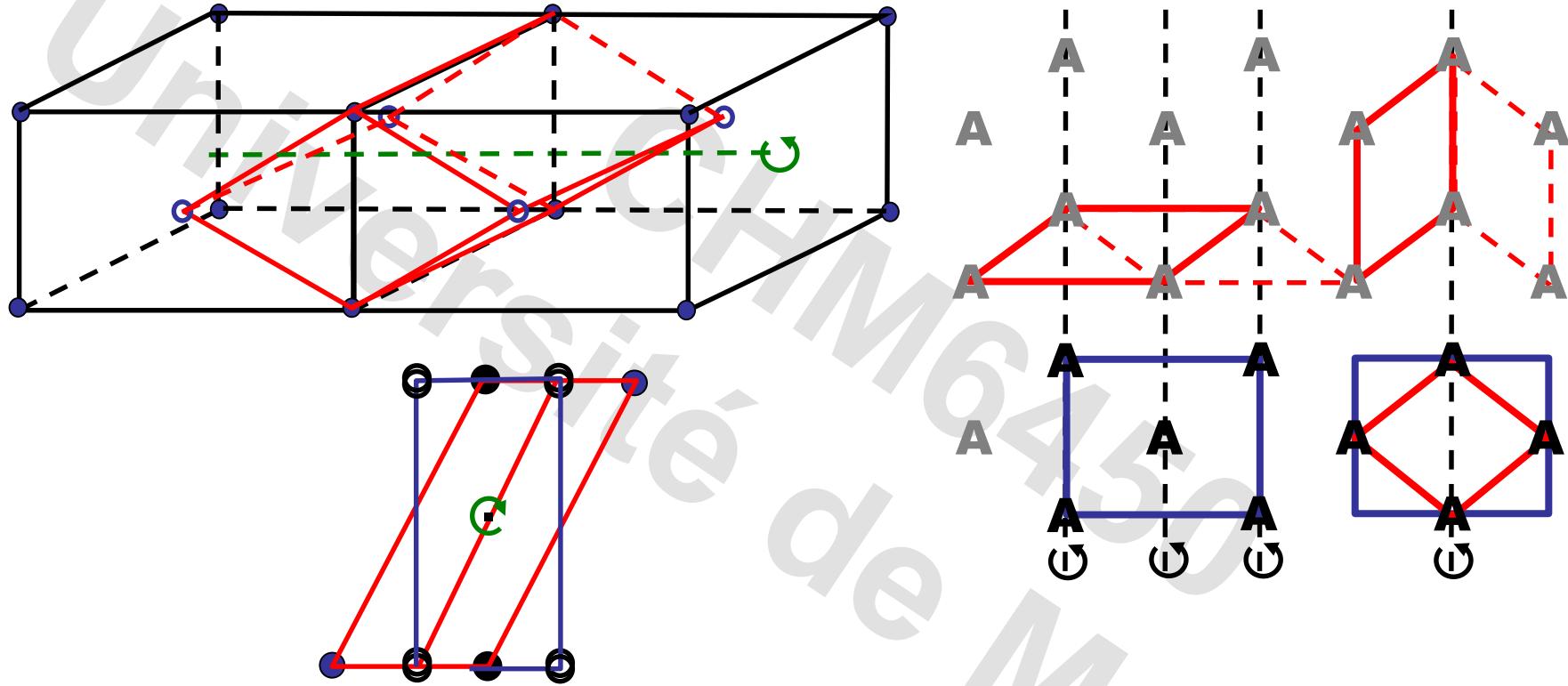


How can we describe a crystal which contains a certain symmetry, for example a C₂ axis or a mirror plane, but the smallest cells are incompatible with these symmetry elements.

We choose a cell of higher volume, containing more than one lattice point, a so called **centered cell**. In this example, we have a centered monoclinic cell.

In crystallographic language, a **face centering C** adds to the existing translations ($x+1, y, z$; $x, y+1, z$; $x, y, z+1$) another one with $x+0.5, y+0.5, z$.

Actually... we do not need them!

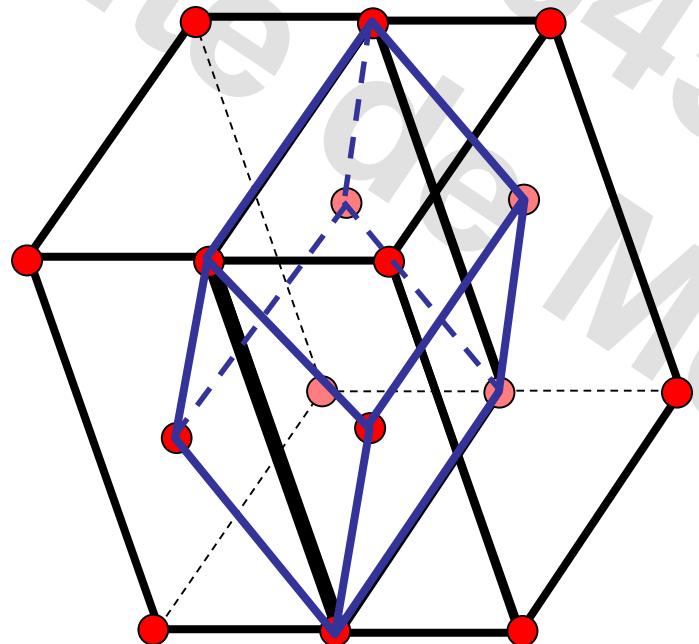


- There is a triclinic cell which is compatible with a C₂ axis or a mirror plane: triclinic with $a = c$ and $\alpha = \gamma$.
- But instead of introducing additional unit cells, centering was introduced.
- Thus instead of 14 crystal systems, we have 7 crystal systems + centering = 14 Bravais lattices

The 14 Bravais lattices

7 crystal systems and 6 centerings: Why do we not have 42 Bravais lattices?

Triclinic: Only P (primitive). Every centered lattice can be transformed into a primitive one.



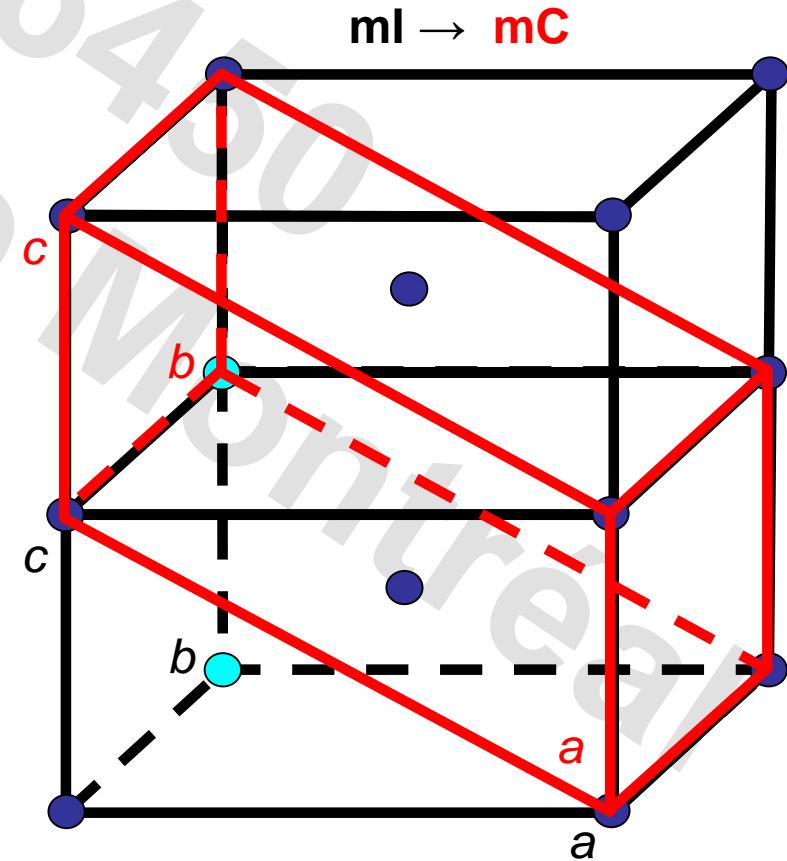
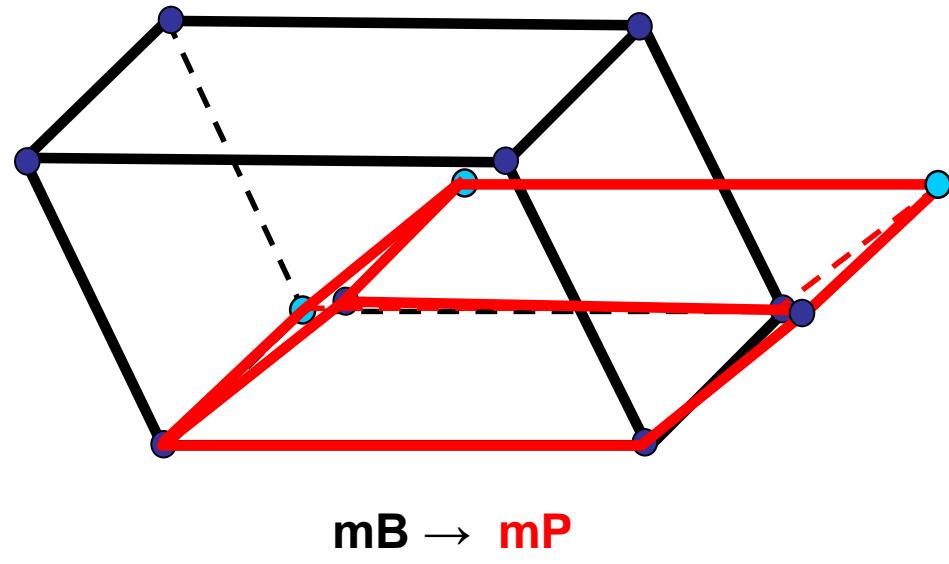
The 14 Bravais lattices

7 crystal systems and 6 centerings: Why do we not have 42 Bravais lattices?

Triclinic: Only aP

Monoclinic: Only mP and mC

- A can be transformed into C by exchanging the axes a and c.
- B can be transformed into P.
- I can be transformed into C.



The 14 Bravais lattices

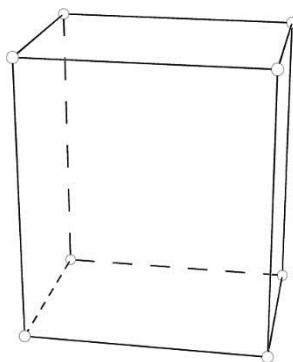
7 crystal systems and 6 centerings: Why do we not have 42 Bravais lattices?

Triclinic: Only aP

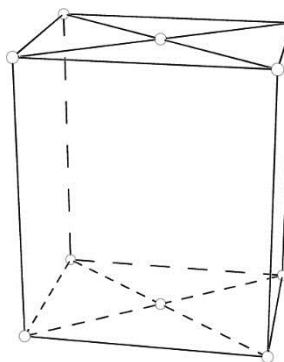
Monoclinic: Only mP and mC

Orthorhombic: oP, oA, ol, oF

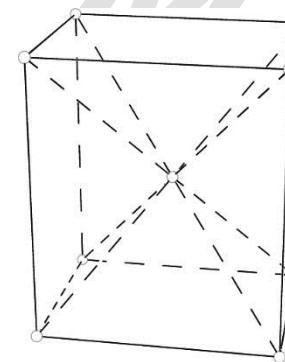
oB and **oC** can be transformed into **oA** by simple axis exchange



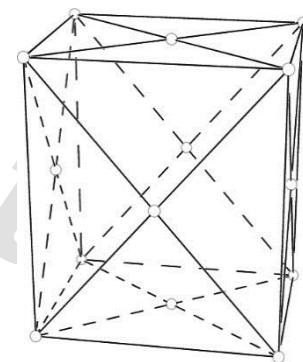
oP



oA



ol



oF

The 14 Bravais lattices

7 crystal systems and 6 centerings: Why do we not have 42 Bravais lattices?

Triclinic: Only aP

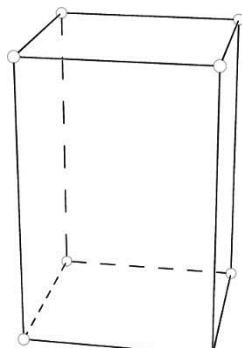
Monoclinic: Only mP and mC

Orthorhombic: oP, oA, ol, oF

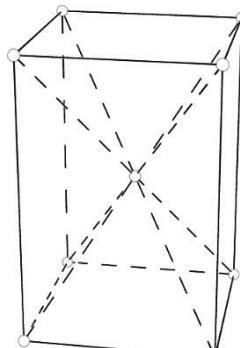
Tetragonal: tP and tl (Because of the C_4 -symmetry, tA becomes tF, which can be transformed into tl. tC can be transformed into tP.)

Trigonal, hexagonal

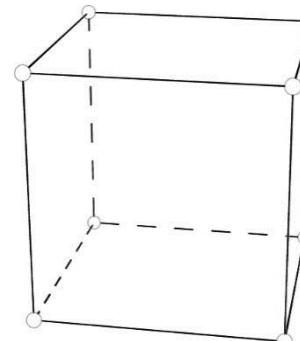
Cubique: cP, cl et cF (C_3 symmetry: cA/cB/cC become automatically cF)



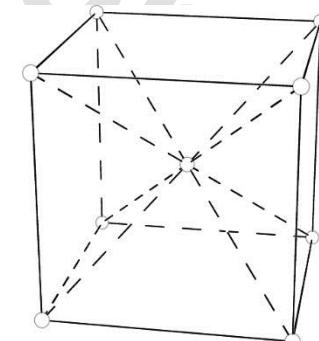
tP



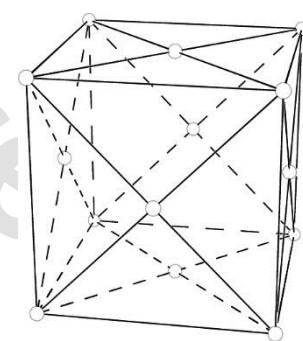
tl



cP



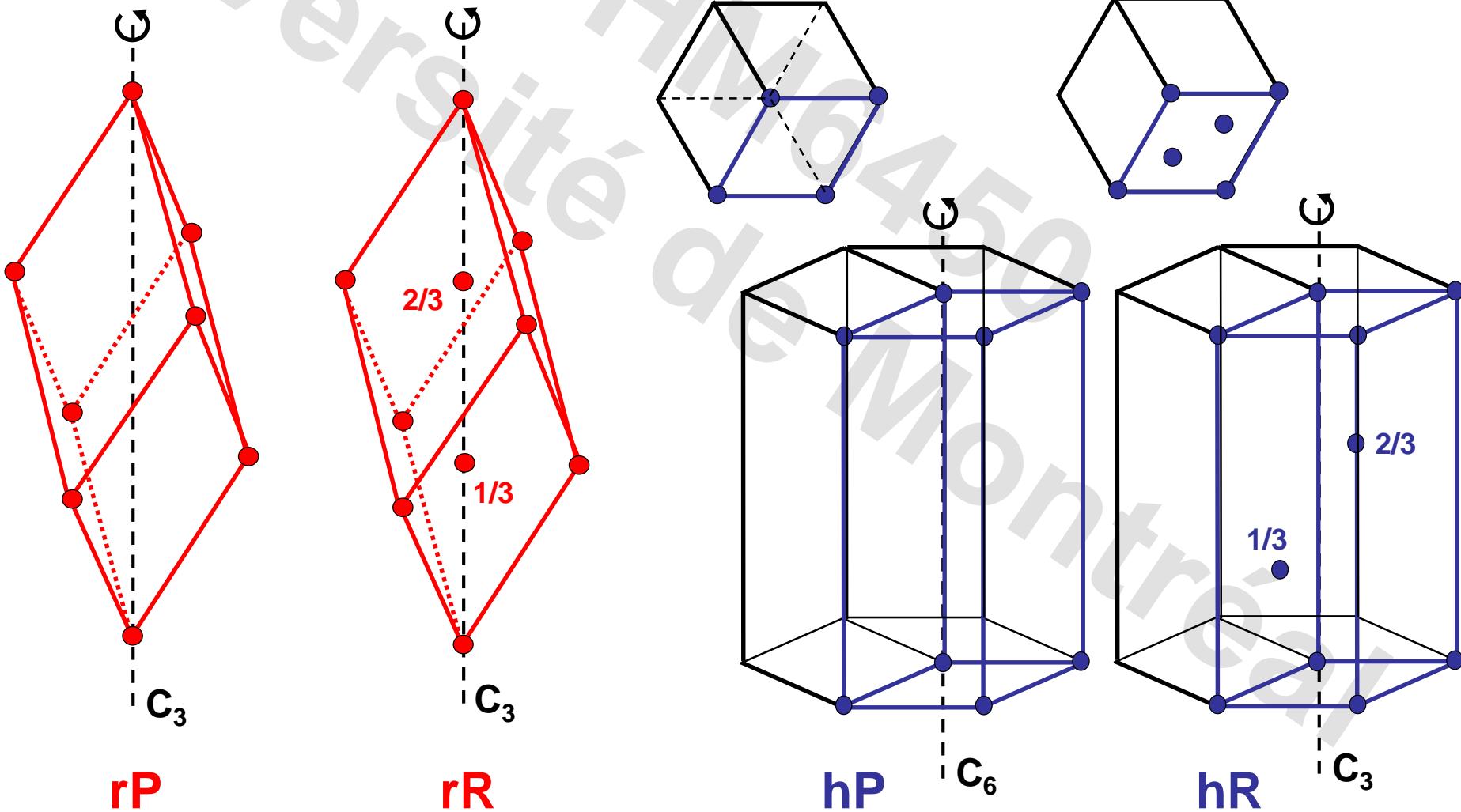
cl



cF

Trigonal, hexagonal, rhomboédrique

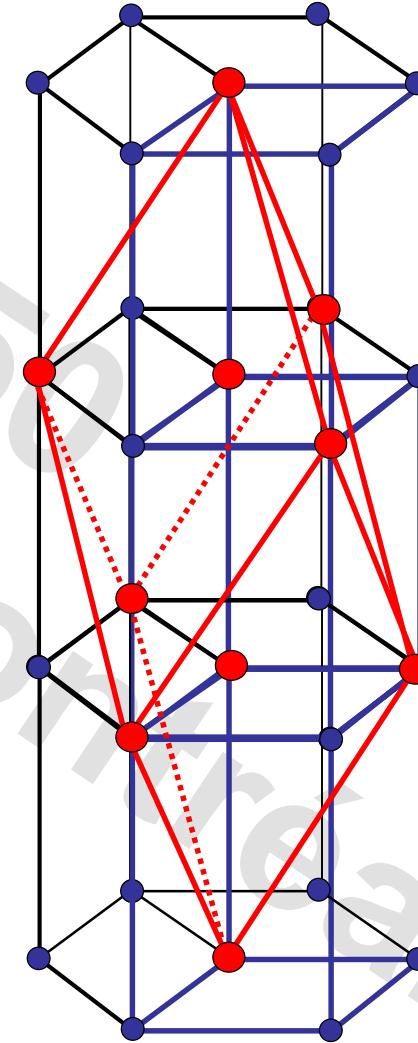
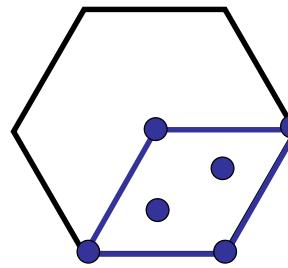
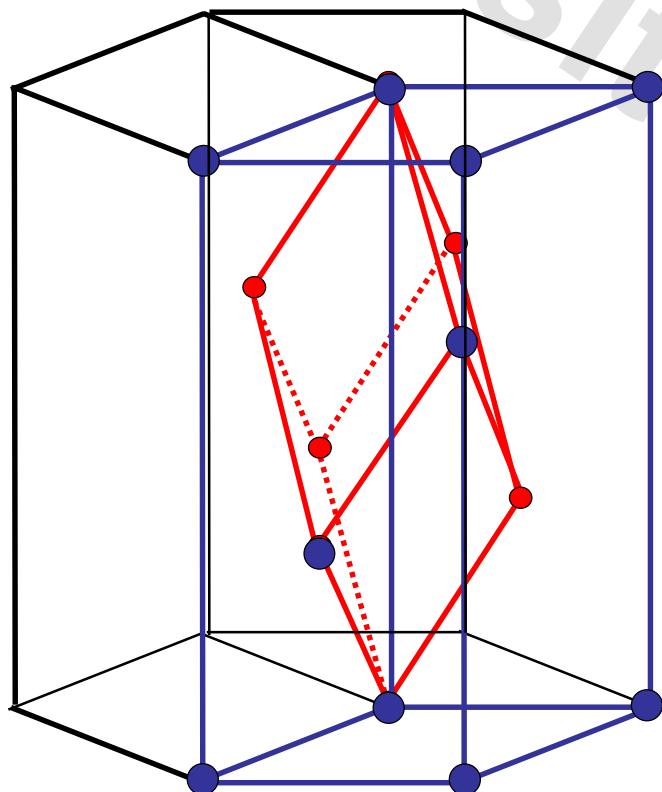
The trigonal/rhombohedral ($a=b=c$, $\alpha=\beta=\gamma\neq 90^\circ$) and hexagonal ($a=b$, $\alpha=\beta=90^\circ$, $\gamma=120^\circ$) crystal systems can only have **rhombohedral centering** to be compatible with C_3 and C_6 symmetry.



Transformation rhomboédrique - hexagonal

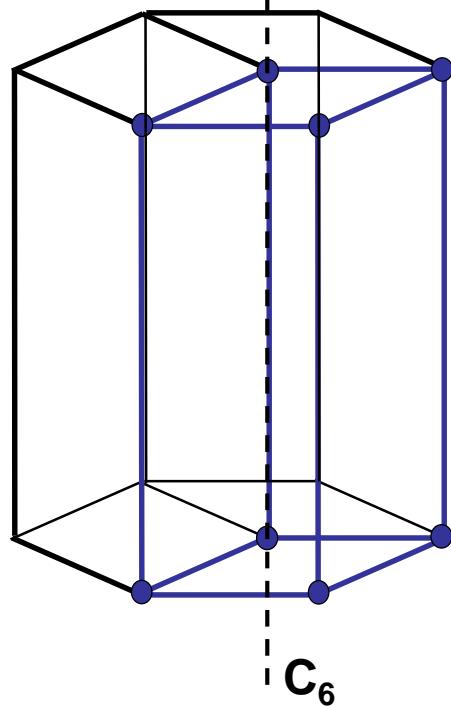
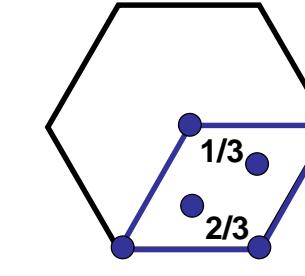
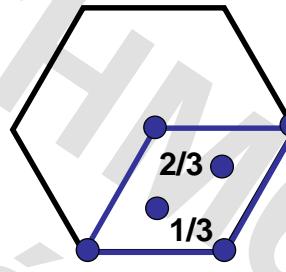
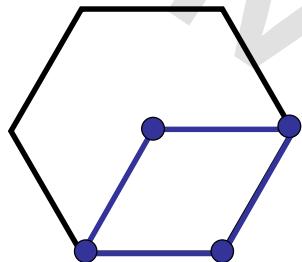
Rhombohedral (trigonal) primitive (**rP**) can be transformed into **hR**.

Hexagonal primitive (**hP**) can be transformed into **rR**.

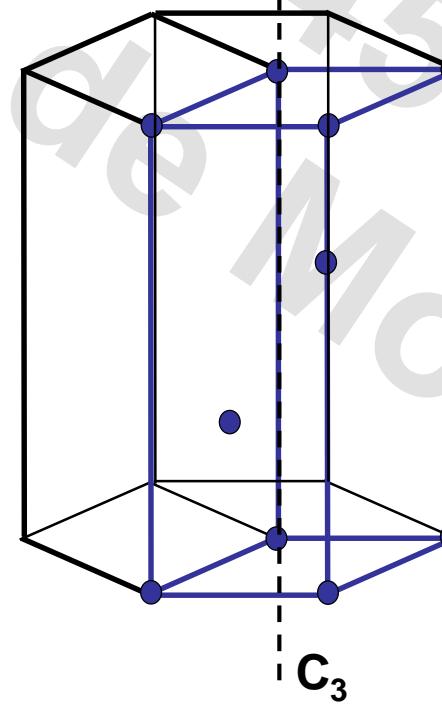


Trigonal, hexagonal, rhomboédrique

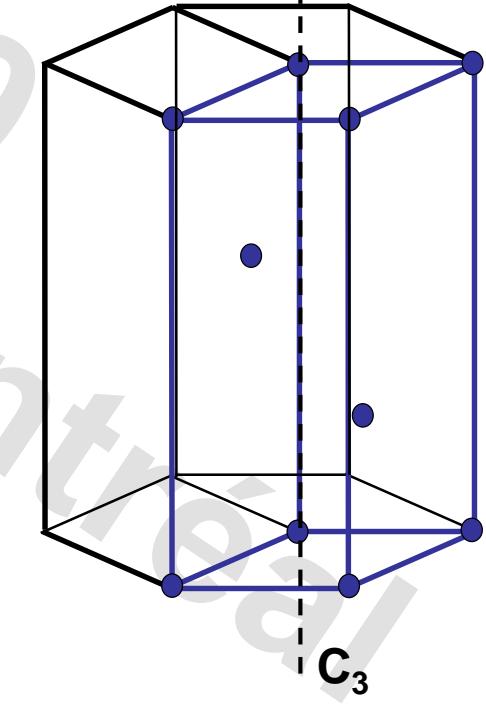
We can thus use the hexagonal unit cells for the trigonal crystal systems, which facilitates understanding and calculation.



hP



hR (obverse)



hR (reverse)

Les 14 réseaux de Bravais («Bravais lattice»)

Triclinic: **aP**

Monoclinic: **mP et mC**

Orthorhombic: **oP, oA, ol, oF**

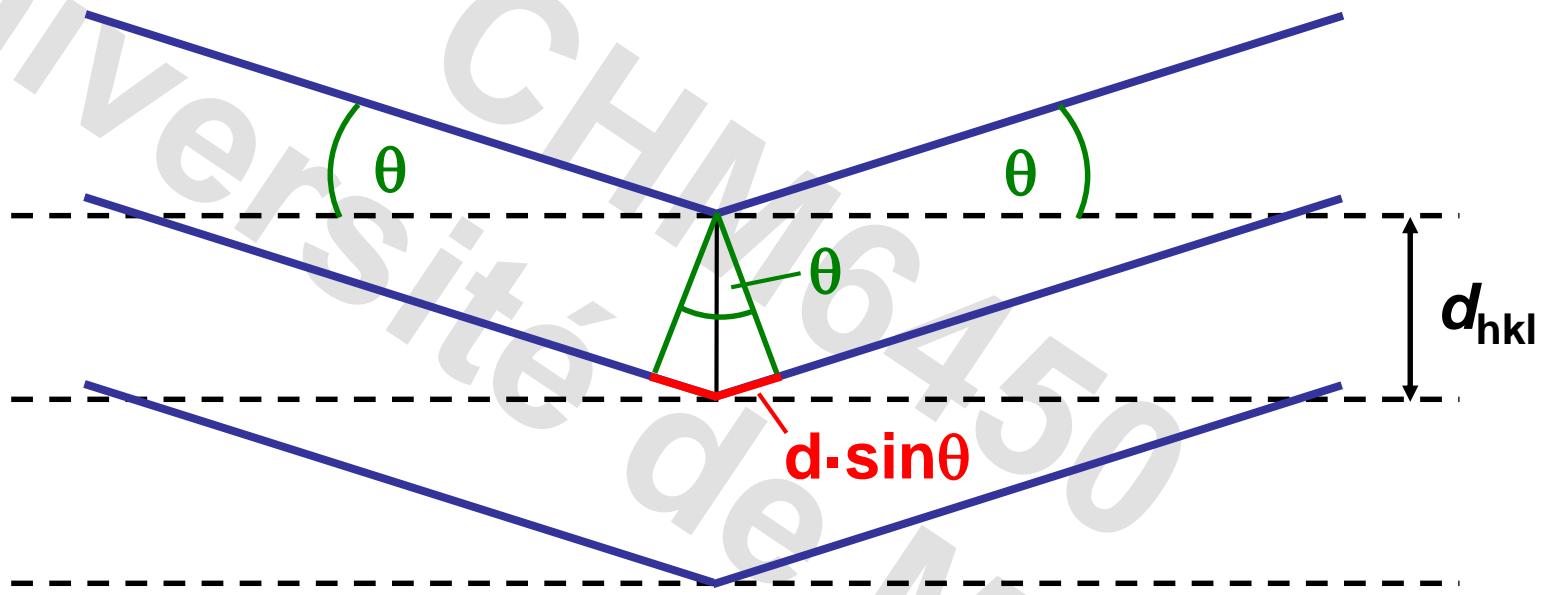
Tetragonal: **tP et tl**

Trigonal, hexagonal: **hP, hR (obverse et reverse)**

Cubic: **cP, cl et cF**

Centered cells have a higher volume than the corresponding primitive cell.

What happens if we increase the size of our unit cell?



Equiangular reflection at the lattice plane hkl of the crystal, which obeys the Laue conditions.

Bragg law:

$$2d_{hkl} \cdot \sin\theta = \lambda \quad (n = 1, 2, 3 \dots)$$

What happens if we increase the size of our unit cell?

Bragg Law:

$$2d_{hkl} \cdot \sin\theta = \lambda \Leftrightarrow \sin\theta = \frac{1}{2} \lambda / d_{hkl}$$

Tetragonal unit cell:

$$a=b=5 \text{ \AA}, c=10 \text{ \AA}$$

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	1	10 \AA	0.077	4°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	2	5 \AA	0.154	9°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	3	3.3 \AA	0.231	13°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	4	2.5 \AA	0.308	18°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	5	2 \AA	0.385	23°

$$a=b=5 \text{ \AA}, c=20 \text{ \AA}$$

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	1	20 \AA	0.039	2°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	2	10 \AA	0.077	4°

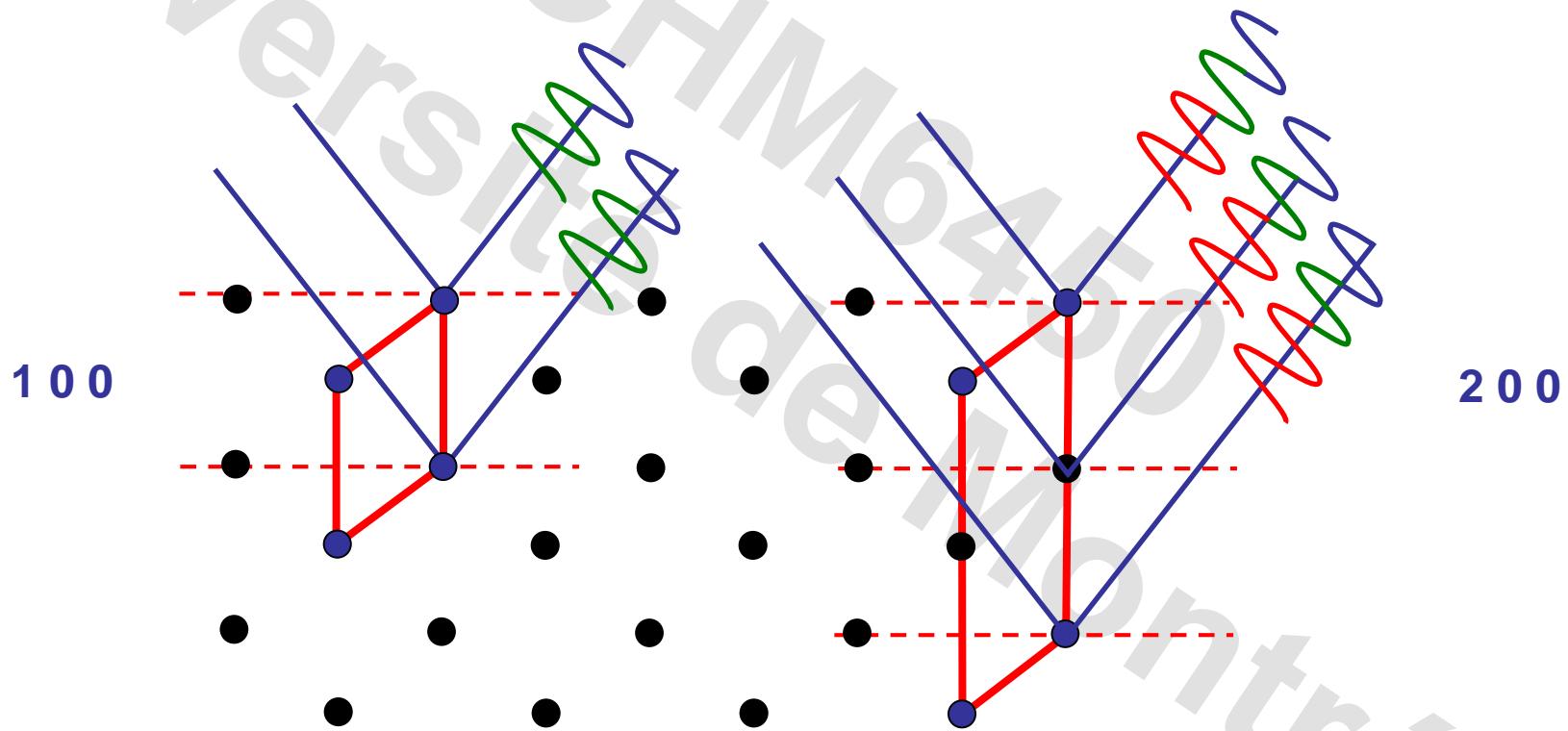
h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	3	6.7 \AA	0.116	7°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	4	5 \AA	0.154	9°

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	5	4 \AA	0.193	11°

What happens if we increase the size of our unit cell?

A unit cell with two times the volume has twice the number of reflections in the same θ region. **Can we thus increase the number of reflections by increasing the size of our unit cell?**



On doubling the axis length a reflection $\{1\ 0\ 0\}$ becomes $\{2\ 0\ 0\}$.

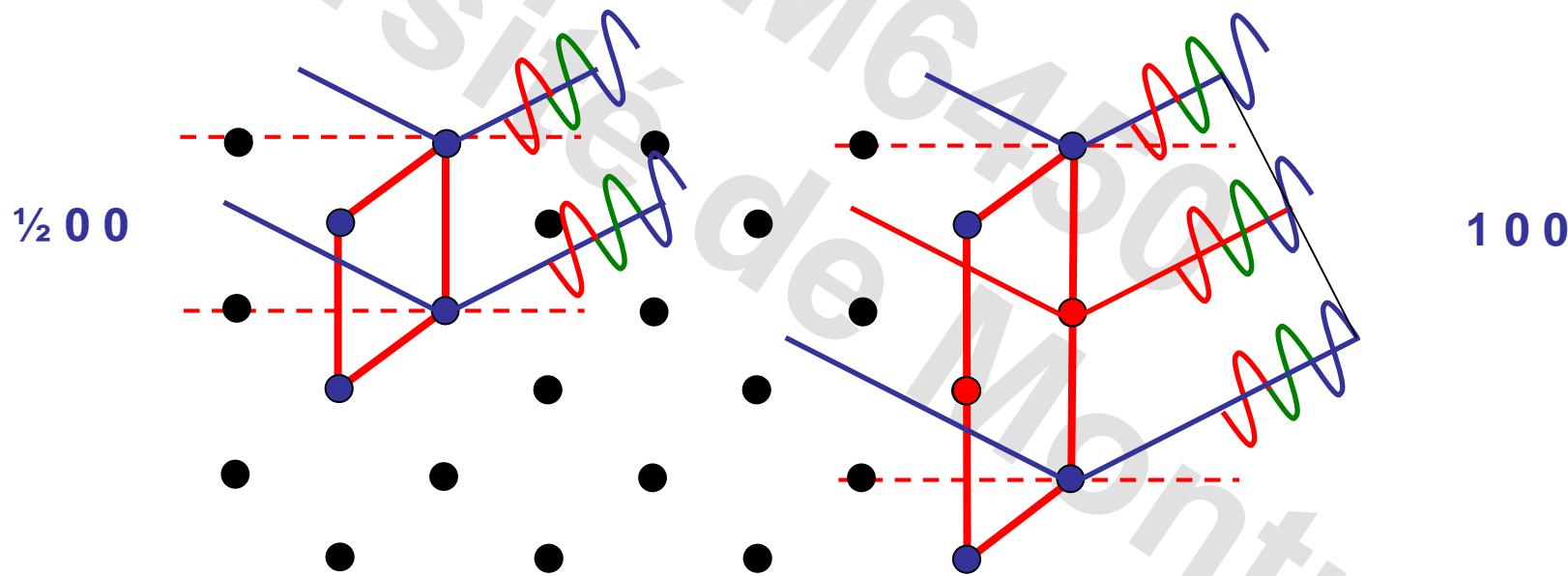
What about the “new” reflection $\{1\ 0\ 0\}$ of our increased unit cell?

What happens if we increase the size of our unit cell?

What about the “new” reflection $\{1\ 0\ 0\}$ of our increased unit cell?

We find systematically another atom at $d_{hkl}/2$. (In other words, we introduced by doubling of the unit cell a new translation operation $x+0.5, y, z.$).

The path length difference to this atom is $\frac{1}{2} \lambda$.



Due to the additional translational symmetry, only reflections with $\{h\ k\ l\}$ with $h = 2n$ are present (path length difference = $2n\lambda$). For reflections $\{h\ k\ l\}$ with odd h , the reflections are **systematically absent**, since the atom $x+0.5$ causes destructive interference.

What happens if we increase the size of our unit cell?

Tetragonal unit cell:

$a=b=5 \text{ \AA}$, $c=10 \text{ \AA}$

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	1	10 \AA	0.077	4°
0	0	2	5 \AA	0.154	9°
0	0	3	3.3 \AA	0.231	13°
0	0	4	2.5 \AA	0.308	18°
0	0	5	2 \AA	0.385	23°

$a=b=5 \text{ \AA}$, $c=20 \text{ \AA}$

h	k	l	d_{hkl}	$\sin\theta$	θ
0	0	1	20 \AA	0.039	2°
0	0	2	10 \AA	0.077	4°
0	0	3	6.7 \AA	0.116	7°
0	0	4	5 \AA	0.154	9°
0	0	5	4 \AA	0.193	11°
0	0	6	3.3 \AA	0.231	13°
0	0	7	2.9 \AA	0.270	16°
0	0	8	2.5 \AA	0.308	18°
0	0	9	2.2 \AA	0.347	20°
0	0	10	2 \AA	0.385	23°
0	0	11	1.8 \AA	0.424	25°

An artificial increase of an axis length adds new reflections, which are, however, systematically absent ($l = 0$).

Systematic absences and centering

The introduction of a translational symmetry ($x+0.5, y, z$) causes the systematic absence of all reflections $\{h k l\}$ with $h \neq 2n$.

All translational symmetries introduce systematic absences. Since all centering introduce additional translations, centering is associated with the presence of systematic absences, which we can use to **determine** the centering from the reflection list.:

	translation	Z	restrictions
P	-	1	-
A	$x, y+0.5, z+0.5$	2	$k+l = 2n$
B	$x+0.5, y, z+0.5$	2	$h+l = 2n$
C	$x+0.5, y+0.5, z$	2	$h+k = 2n$
F	A + B + C	4	$h+k = 2n, h+l = 2n, k+l = 2n$
I	$x+0.5, y+0.5, z+0.5$	2	$h+k+l = 2n$

Thus from investigating systematic absences in the reflection list, we can determine the Bravais lattice.

Systematic absences

In the same way, symmetry elements which include translations also cause systematic absences:

Screw axes (axes hélicoïdal):

		translation	restrictions
$2_1, 4_2, 6_3$	a	$x+0.5$	$h00$ avec $h = 2n$
	b	$y+0.5$	$0k0$ avec $k = 2n$
	c	$z+0.5$	$00l$ avec $l = 2n$
$3_1, 3_2, 6_2, 6_4$	c	$z+1/3$	$00l$ avec $l = 3n$
$4_1, 4_3$	c	$z+0.25$	$00l$ avec $l = 4n$
$6_1, 6_5$	c	$z+1/6$	$00l$ avec $l = 6n$

Systematic absences

Glide planes:

		translation	zonal restrictions
b	$\perp a$	$y+0.5$	$0kl$ avec $k = 2n$
c	$\perp a$	$z+0.5$	$0kl$ avec $l = 2n$
n	$\perp a$	$y+0.5, z+0.5$	$0kl$ avec $k+l = 2n$
d	$\perp a$	$y+0.25, z+0.25$	$0kl$ avec $k+l = 4n$ (F)
c	$\perp b$	$z+0.5$	$h0l$ avec $l = 2n$
a	$\perp c$	$x+0.5$	$hk0$ avec $h = 2n$
c	$\perp [110]$	$z+0.5$	hhl avec $l = 2n$ (t, c)
c	$\perp [120]$	$z+0.5$	hhl avec $l = 2n$ (trigonal)
d	$\perp [110]$	$x+0.5, z+0.25$	hhl avec $2h+l = 4n$ (t, cl)

Space groups

The combination of the translations, defined by the Bravais lattice, and the elements of symmetry possible in an infinite crystal result in the 230 possible space groups. (Group theory: a space group must be closed, i. e. the action of a new element cannot generate an element which is not already in the group.)

Example: The most common space group: P2₁/c

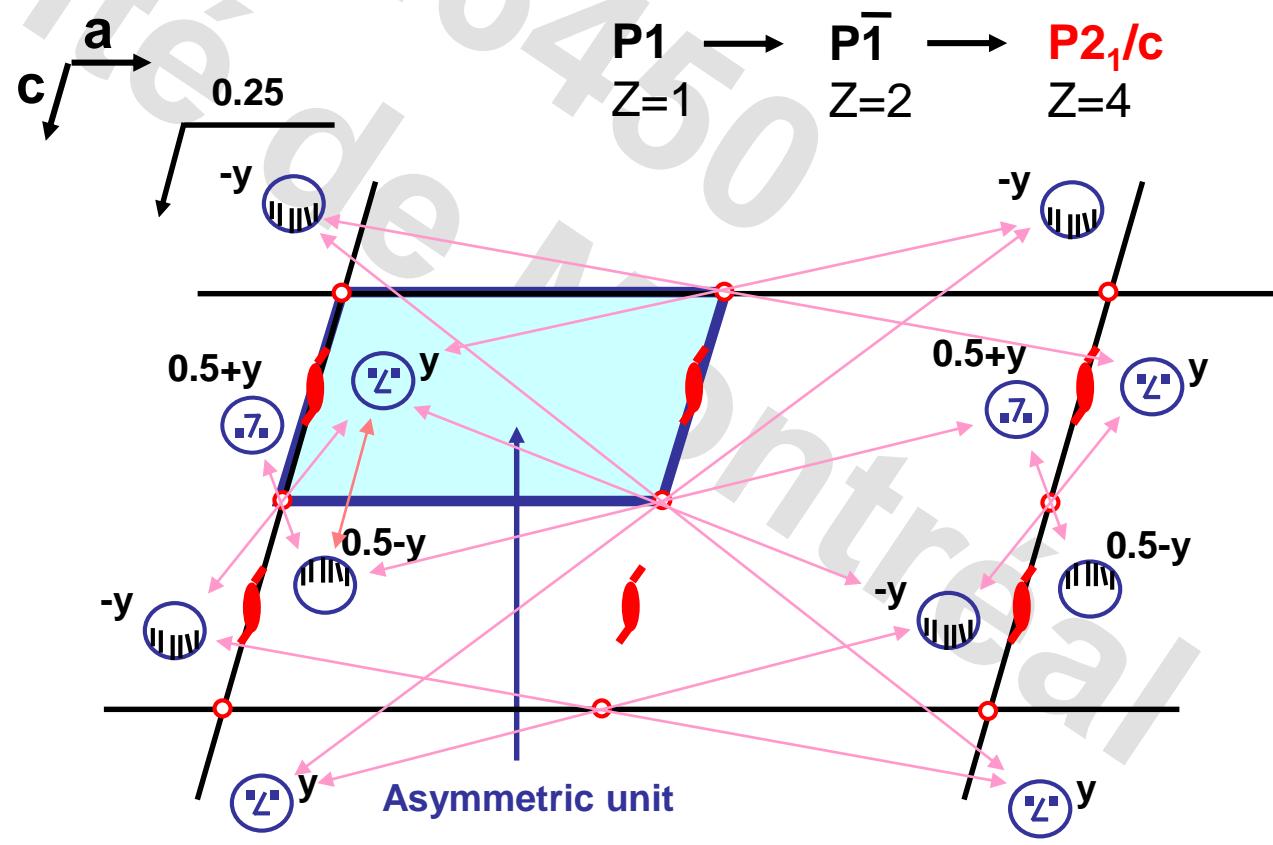
x+1, y, z
x, y+1, z
x, y, z+1

-x, -y, -z *i*

0.5-x, 0.5-y, 0.5-z *i*
0.5-x, y, z *i*
x, 0.5-y, z *i*
x, y, 0.5-z *i*
x, 0.5-y, 0.5-z *i*
0.5-x, y, 0.5-z *i*
0.5-x, 0.5-y, z *i*

x, 0.5-y, z+0.5 *c*

-x, 0.5+y, 0.5-z *2₁*



Space group determination

The determination of the correct space group is essential since without a correct description of the symmetry refinement of a structure is not possible (overparametrization or lack of parameters).

To determine the space group we can use the following information:

- The Laue group (Symmetry of the reflections)
- The presence of screw axes and glide planes (systematic absences)
- The presence of an inversion center (value E(E-1), chirality of the molecule)

Crystal systems and Laue group

The combination of the crystal systems with allowed symmetry elements yields 32 crystal classes (or crystallographic point groups):

Crystal system	Crystal class	Crystal system	Crystal class
Triclinic	C_1 1 C_i -1	Trigonal	C_3 3 C_{3i} -3
Monoclinic	C_2 2 C_s m C_{2h} 2/m		D_3 32 C_{3v} 3m D_{3d} -3m
Orthorhombic	D_2 222 C_{2v} mm2 D_{2h} mmm	Hexagonal	C_6 6 C_{3h} -6 C_{6h} 6/m
Tetragonal	C_4 4 S_4 -4 C_{4h} 4/m D_4 422 C_{4v} 4mm D_{2d} -42m D_{4h} 4/mmm	Cubic	D_6 622 C_{6v} 6mm D_{3h} -6m2 D_{6h} 6/mmm
			T 23 T_h m3 O 432 T_d -43m O_h m3m

The Laue group results from the combination of crystal class and Friedel's Law

Friedel law

We know that the structure factor is given by:

$$F_{hkl} = \sum_{j=1}^N f_j \cdot e^{2\pi i(h \cdot x_j + k \cdot y_j + l \cdot z_j)} = \sum_{j=1}^N f_j \cdot e^{i\alpha_j(hkl)}$$

For the inverse reflection (-h -k -l) we obtain: $\alpha_j(\bar{h}\bar{k}\bar{l}) = 2\pi(-h \cdot x_j - k \cdot y_j - l \cdot z_j) = -\alpha_j(hkl)$

$$F_{\bar{h}\bar{k}\bar{l}} = \sum_{j=1}^N f_j \cdot e^{i\alpha_j(\bar{h}\bar{k}\bar{l})} = \sum_{j=1}^N f_j \cdot e^{-i\alpha_j(hkl)}$$

$$|e^{i\varphi}|^2 = \cos^2 \varphi + \sin^2 \varphi; \quad \cos(-\varphi) = \cos(\varphi); \quad \sin(-\varphi) = -\sin(\varphi)$$

$$|e^{-i\varphi}|^2 = \cos^2(-\varphi) + \sin^2(-\varphi) = \cos^2 \varphi + (-\sin \varphi)^2 = |e^{i\varphi}|^2$$

$$\Rightarrow I_{hkl} = |F_{hkl}|^2 = I_{\bar{h}\bar{k}\bar{l}}$$

Our reciprocal lattice is thus always centrosymmetric!

32 crystal classes, but 11 Laue groups

Adding the inversion symmetry caused by Friedel's law, we obtain 11 Laue groups. **The Laue group describes the symmetry of our observed reflections!**

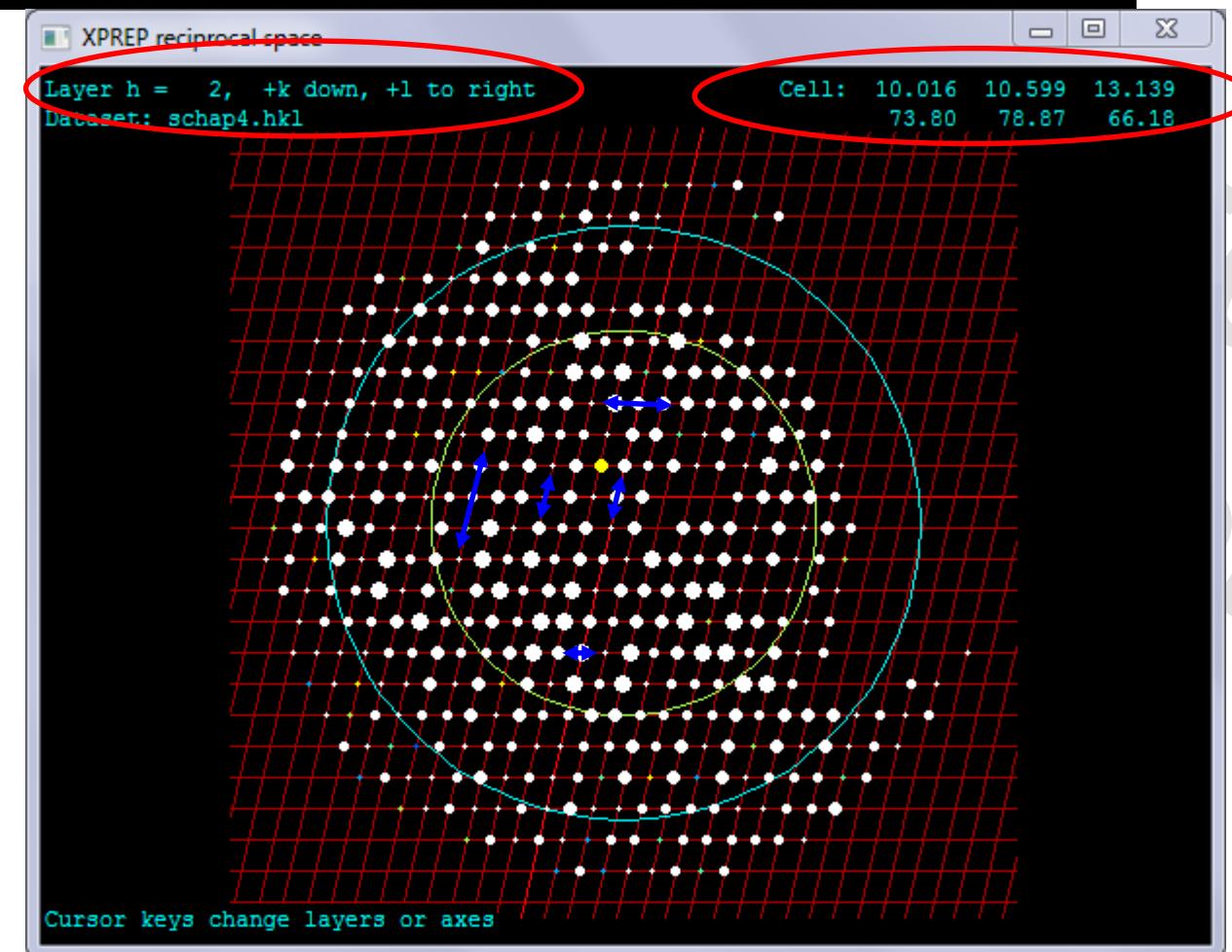
Crystal system	Crystal class	Laue group
Triclinic	C_1	1
	C_i	-1
Monoclinic	C_2	2
	C_s	m
	C_{2h}	2/m
Orthorhombic	D_2	222
	C_{2v}	mm2
	D_{2h}	mmm
Tetragonal	C_4	4
	S_4	-4
	C_{4h}	4/m
	D_4	422
	C_{4v}	4mm
	D_{2d}	-42m
	D_{4h}	4/mmm

Crystal system	Crystal class	Laue group
Trigonal	C_3	3
	C_{3i}	-3
	D_3	32
Hexagonal	C_{3v}	3m
	D_{3d}	-3m
	C_6	6
Cubic	C_{3h}	-6
	C_{6h}	6/m
	D_6	622
	C_{6v}	6mm
	D_{3h}	-6m2
	D_{6h}	6/mmm
	T	23
	T_h	m3
	O	432
	T_d	-43m
	O_h	m3m

Visualisation of reciprocal space with XPREP

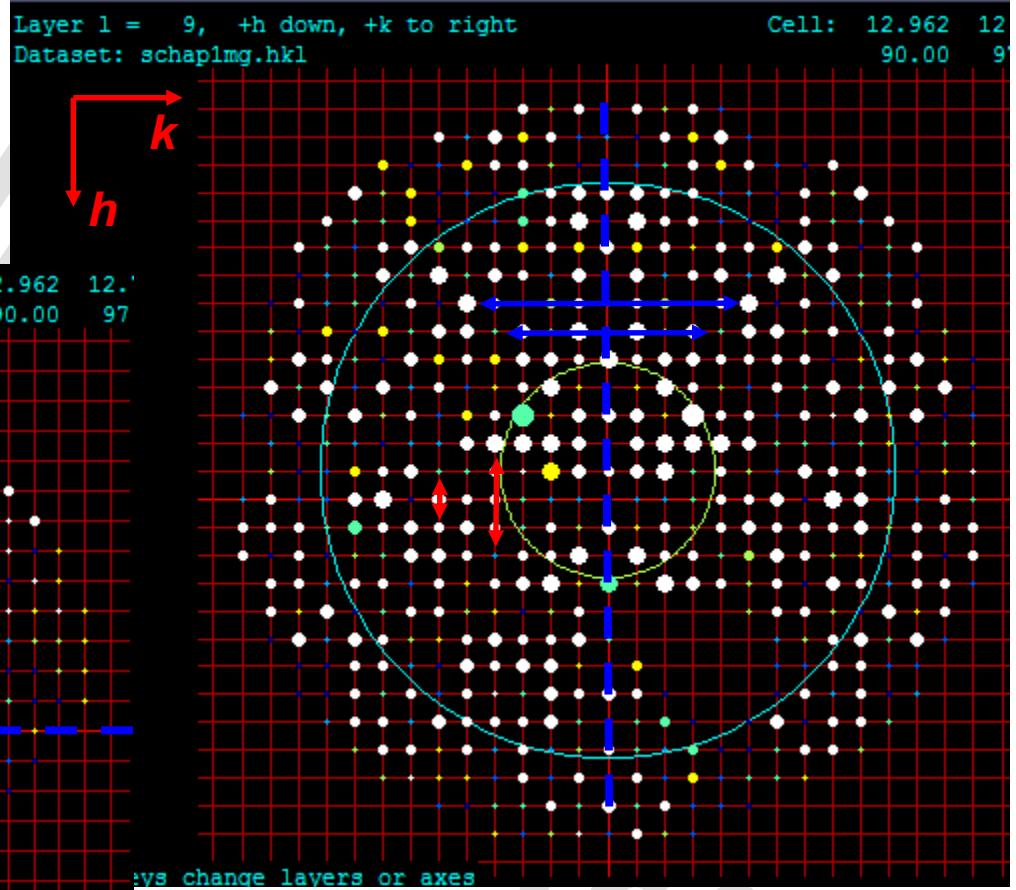
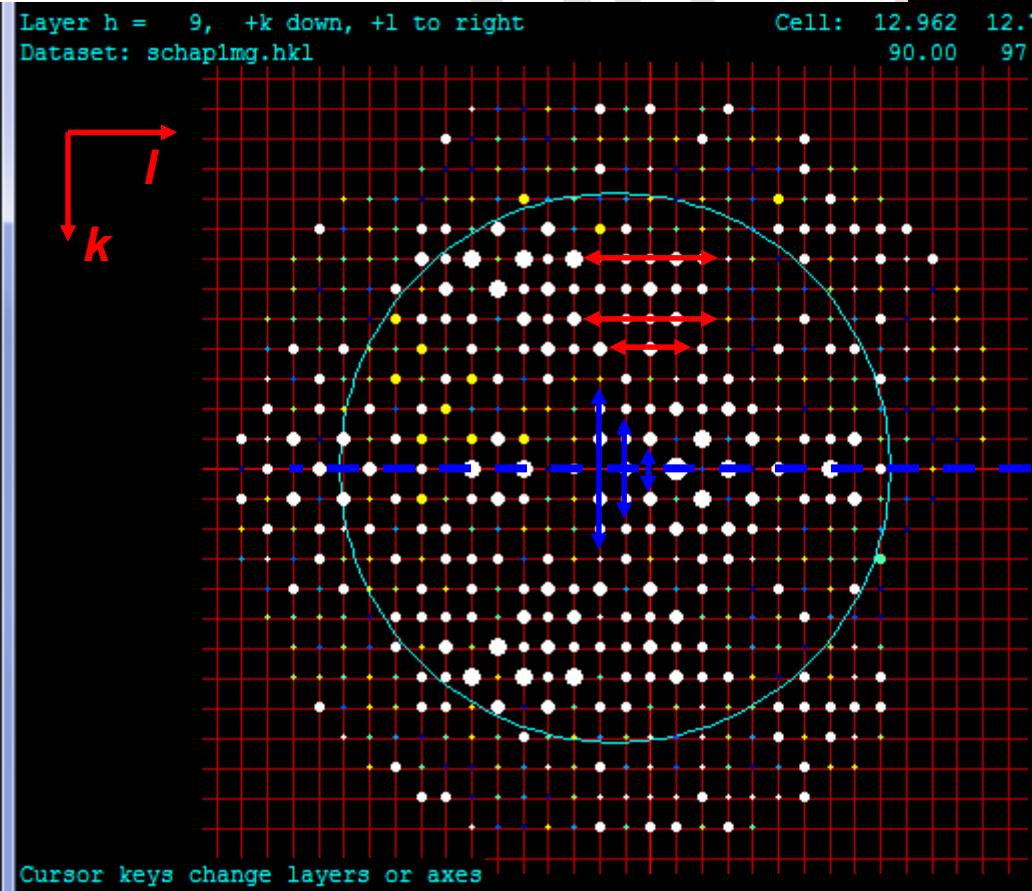
```
[D] Read, modify or merge DATASETS      [C] Define unit-cell CONTENTS
[P] Contour PATTERSON sections          [F] Set up shelxtl FILES
[H] Search for HIGHER metric symmetry   [R] RECIPROCAL space displays
[S] Determine or input SPACE GROUP       [U] UNIT-CELL transformations
[A] Absorption, powder, SIR, SAD, MAD etc. [T] Change TOLERANCES
[M] Test for MEROHEDRAL TWINNING        [O] Self-rotation function
[L] Reset LATTICE type of original cell  [Q] QUIT program
```

Select option [D]: █



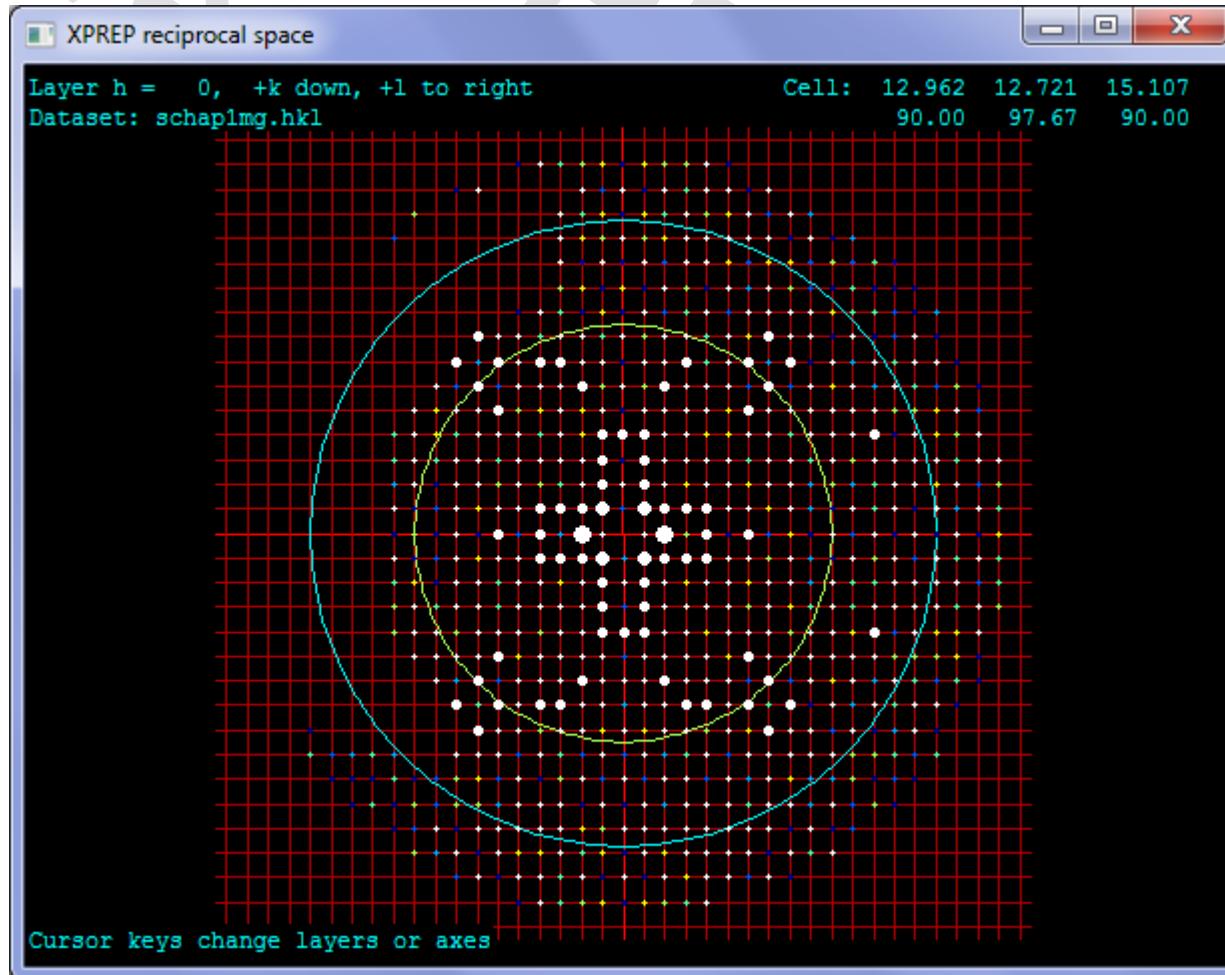
Triclinic:
No symétrie but
inversion
(Friedel's Law)

Monoclinic: $hkl = h -k l$, mirror plane perpendicular to k

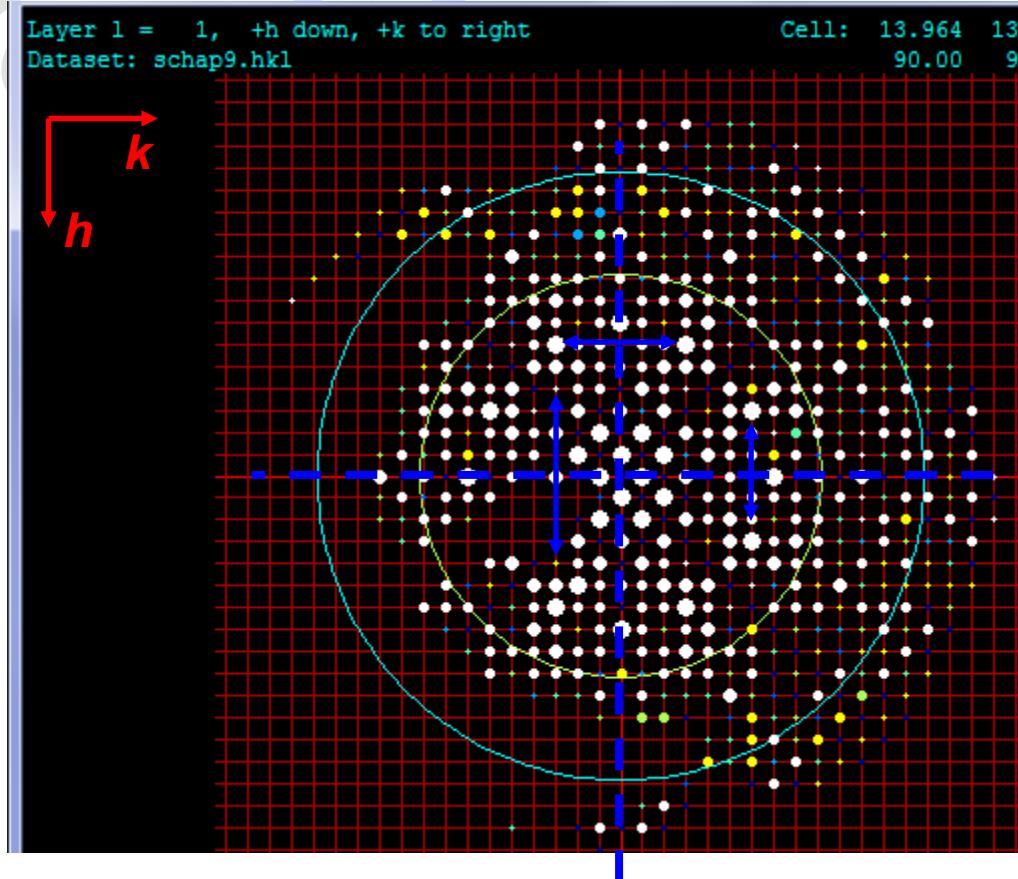


Monoclinic: $hkl = h -k l$, mirror plane perpendicular to k

Avoid planes with $h=0$, $k=0$ or $l=0$, since $h = -h$ is trivial for $h = 0$



Tetragonal: Two Laue groups: 4/m or 4/mmm:
4-fold axis parallel to l , either mirror plane perpendicular to l or at all axes



Here: 4/mmm

Space group determination

The determination of the correct space group is essential since – without a correct description of the symmetry – neither the solution nor the refinement of a structure is possible..

To determine the space group we can use the following information:

- The Laue group (Symmetry of the reflections)
- The presence of screw axes and glide planes (systematic absences)
- The presence of an inversion center (value E(E-1), chirality of the molecule)

Systematic absences

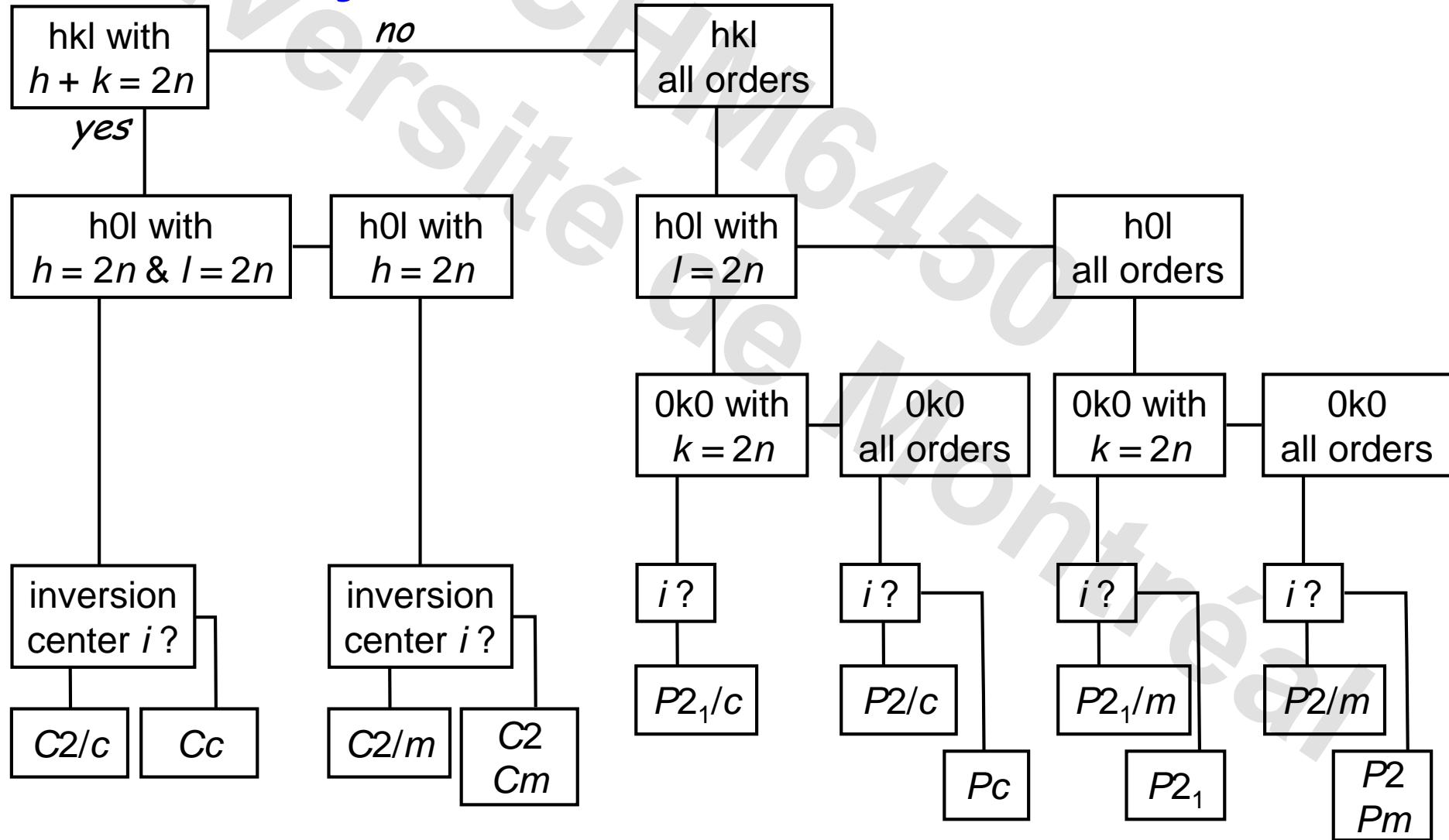
- Determination which screw axes and glide planes are present.
- Determination which space groups contain these symmetry elements.
- By hand
 - Use the provided tables (Studium)
 - Start always at the end of a table (most special case) and work towards the more general ones
 - Check if the reflection condition mentioned is violated. I. e. $h00: h = 2n$: check if there are several reflections $h00$ with odd h , which have intensities higher than 3σ . About 10% of errors are allowed. Thus if in 20 systematic absences, you find two which are $>3\sigma$, the reflection condition might be nevertheless valid.
 - Once you find which table A, B, C... you are in, go to the subtable. Start again at the end.

Space group determination by hand

Example Vivan8:

$a = 30.8391(7)$ Å, $b = 14.2009(4)$ Å, $c = 29.0930(7)$ Å, $\alpha = 90^\circ$, $\beta = 119.0730(10)^\circ$, $\gamma = 90^\circ$

Test for C centering

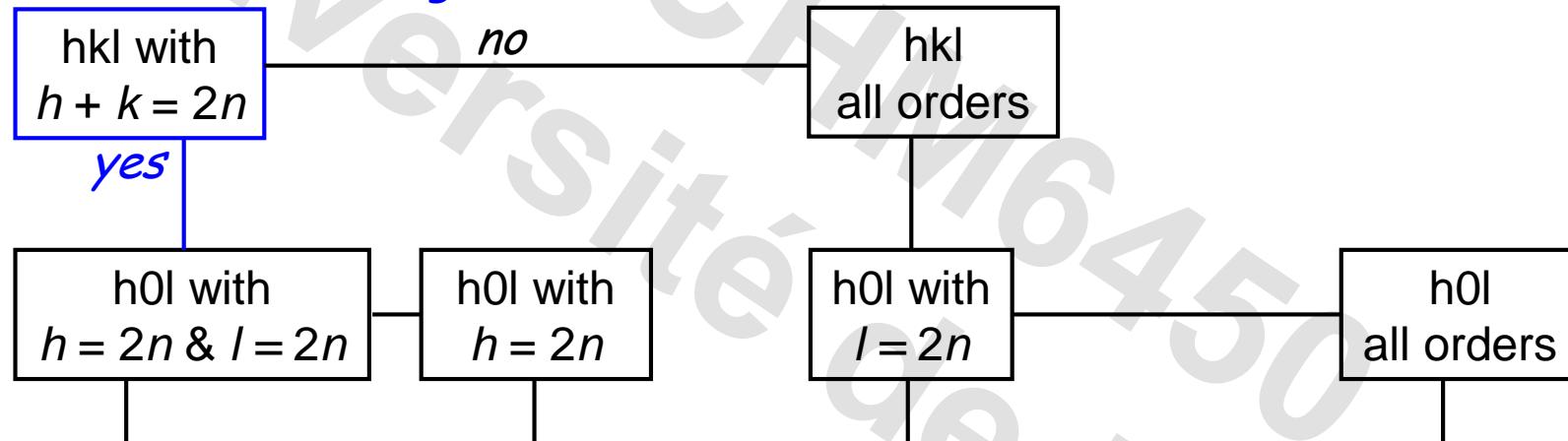


Space group determination by hand

Example Vivan8:

$a = 30.8391(7)$ Å, $b = 14.2009(4)$ Å, $c = 29.0930(7)$ Å, $\alpha = 90^\circ$, $\beta = 119.0730(10)^\circ$, $\gamma = 90^\circ$

Test for C centering



hkl all orders: 127437

hkl with $k+l \neq 2n$: 20365 of 63653

hkl with $h+l \neq 2n$: 20487 of 63704

hkl with $h+k \neq 2n$: 0 of 63827

hkl with $h+k+l \neq 2n$: 20319 of 63810

hkl with $k+l \neq 2n$, $h+l \neq 2n$, $k+l \neq 2n$: 27797 of 95592

inversion center i

$C2/c$

↓

↓

Cm

↓

Pc

↓

$P2_1$

↓

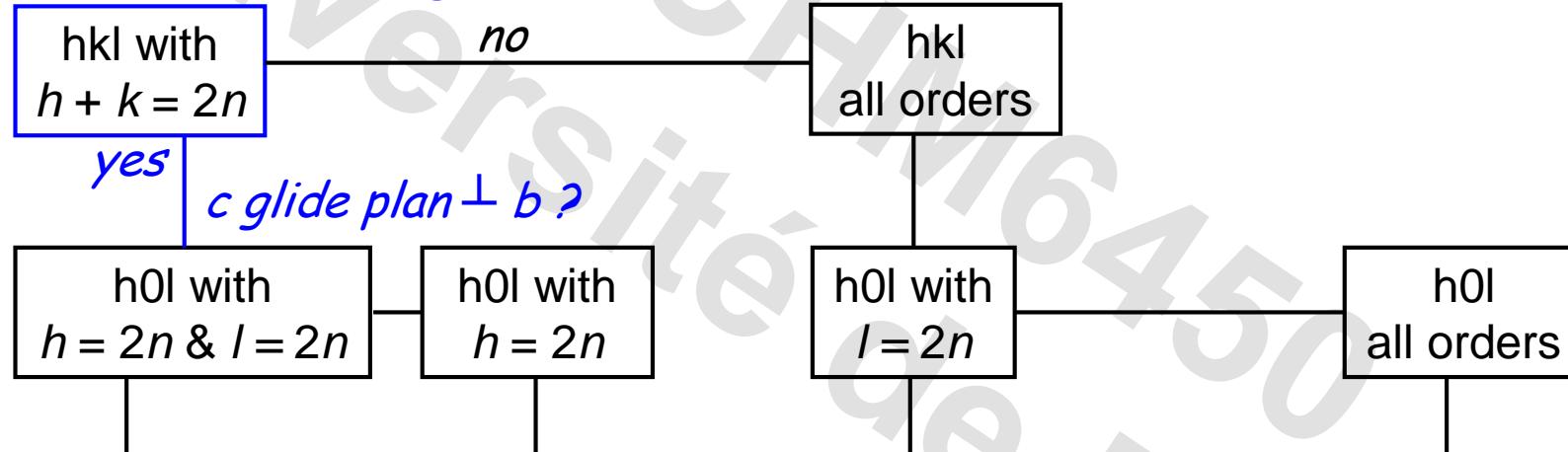
$P2$
 Pm

Space group determination by hand

Example Vivan8:

$a = 30.8391(7)$ Å, $b = 14.2009(4)$ Å, $c = 29.0930(7)$ Å, $\alpha = 90^\circ$, $\beta = 119.0730(10)^\circ$, $\gamma = 90^\circ$

Test for C centering



h	k	l	I	σ	0	0	-1	0	0	0.01	0	0	-17	-0.04	0.1	1	0	5	0.04	0.04	2	0	9	-0.04	0.04		
0	0	-1			0	0	-17	-0.04	0.1		1	0	5	0.04	0.04	2	0	9	-0.04	0.04							
0	0	-2			0.4	0.02		0	0	-18	0.37	0.11	1	0	4	-0.02	0.03	2	0	8	1.34	0.07					
0	0	-3			-0.01	0.02		1	0	19	-0.09	0.1	1	0	3	0.06	0.02	2	0	7	-0.03	0.04					
0	0	-4			127	0.47		1	0	18	-0.17	0.09	1	0	2	0	0.02	2	0	6	100.11	0.41					
0	0	-5			-0.02	0.03		1	0	17	0.02	0.11	1	0	1	0.02	0.01	2	0	5	-0.04	0.04	3	0	12	-0.05	0.08
0	0	-6			10.16	0.11		1	0	16	-0.13	0.09	1	0	0	0	0.01	2	0	4	209.88	0.78	3	0	11	1.52	0.11
0	0	-7			-0.03	0.04		1	0	15	0.69	0.13	2	0	19	-0.11	0.11	2	0	3	-0.01	0.03	3	0	10	0	0.05
0	0	-8			269.59	0.97		1	0	14	-0.01	0.09	2	0	18	13.37	0.28	2	0	2	17.97	0.11	3	0	9	-0.03	0.05
0	0	-9			-0.03	0.04		1	0	13	0.18	0.1	2	0	17	-0.07	0.09	2	0	1	-0.01	0.02	3	0	8	-0.01	0.04
0	0	-10			0.75	0.07		1	0	12	0	0.06	2	0	16	19.37	0.39	2	0	0	251.03	1.01	3	0	7	0.02	0.04
0	0	-11			0.02	0.05		1	0	11	1.49	0.09	2	0	15	-0.11	0.09	3	0	18	-0.06	0.11	3	0	6	-0.01	0.04
0	0	-12			449.53	1.41		1	0	10	-0.06	0.04	2	0	14	20.21	0.36	3	0	17	-0.02	0.1	3	0	5	0.16	0.04
0	0	-13			-0.07	0.06		1	0	9	0	0.04	2	0	13	-0.12	0.08	3	0	16	-0.09	0.11	3	0	4	-0.02	0.04
0	0	-14			25.88	0.38		1	0	8	-0.02	0.04	2	0	12	127.11	0.69	3	0	15	-0.04	0.1	3	0	3	0.45	0.04
0	0	-15			-0.12	0.08		1	0	7	0.84	0.06	2	0	11	-0.09	0.05	3	0	14	-0.02	0.1	3	0	2	0	0.03
0	0	-16			135.75	0.98		1	0	6	-0.03	0.04	2	0	10	26.69	0.23	3	0	13	0.05	0.09	3	0	1	0.13	0.03

0k0
orders



/m

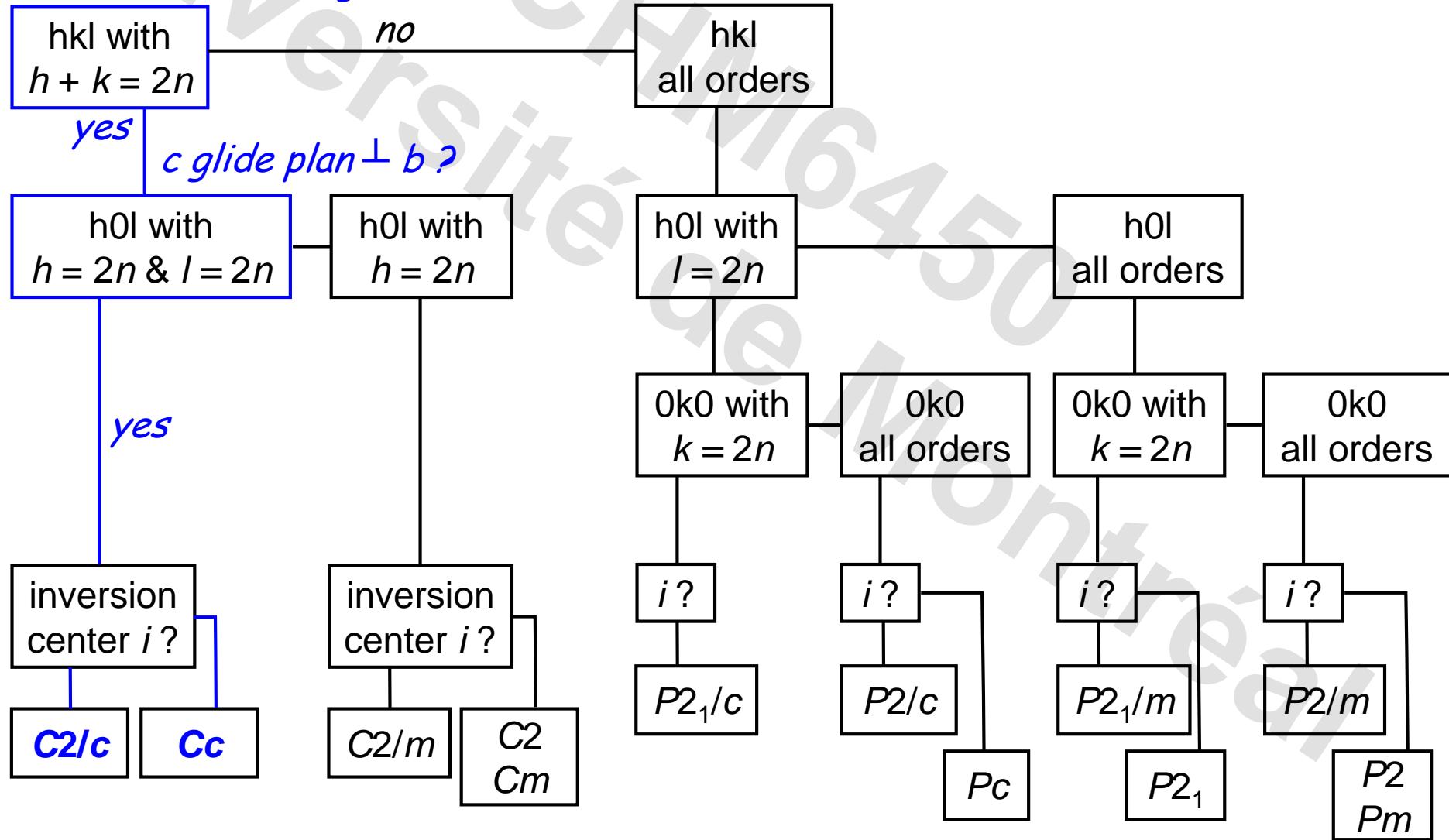
P2
Pm

Space group determination by hand

Example Vivan8:

$a = 30.8391(7)$ Å, $b = 14.2009(4)$ Å, $c = 29.0930(7)$ Å, $\alpha = 90^\circ$, $\beta = 119.0730(10)^\circ$, $\gamma = 90^\circ$

Test for C centering

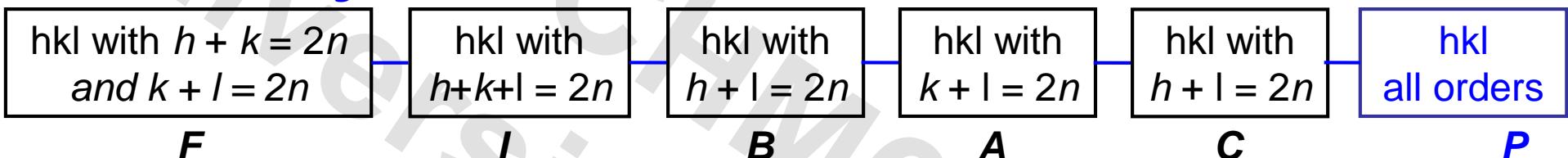


Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering



Lattice exceptions	P	A $k+l=2n$	B $h+l=2n$	C $h+k=2n$	I $h+k+l=2n$	F $k+l, h+l, h+k=2n$	All
Total no. Reflections	0	18455	18302	18397	18426	27577	36770
Total with $I > 3\sigma(I)$	0	14415	14946	14937	14677	22149	30028
Mean $I/\sigma(I)$ (all)	0.0	10.1	10.5	10.5	10.3	10.3	10.5
Mean $I/\sigma(I)$ $I > 3\sigma(I)$	0.0	12.6	12.6	12.7	12.7	12.6	12.6

Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

hkl with $h + k = 2n$ and $k + l = 2n$	hkl with $h+k+l = 2n$	hkl with $h + l = 2n$	hkl with $k + l = 2n$	hkl with $h + k = 2n$	hkl all orders
F	I	B	A	C	P
0 0 1 0,01 0,01					
0 0 2 57,86 1,17					
0 0 3 -0,01 0,02	0 2 1 0,02 0,01				
0 0 4 48,76 0,99	0 2 2 12,96 0,17				
0 0 5 0 0,03	0 2 3 0,02 0,01				
0 0 6 26,39 0,74	0 2 4 96,14 1,37	0 3 10 -0,04 0,05	0 5 5 20,89 0,33	0 6 10 8,95 0,26	0 8 5 -0,02 0,04
0 0 7 -0,06 0,07	0 2 5 0 0,02	0 4 0 6,19 0,13	0 5 6 0 0,05	0 7 0 0,03 0,03	0 8 6 2,07 0,08
0 0 8 12,9 0,8	0 2 6 57,37 1,18	0 4 1 0,01 0,02	0 5 7 20,26 0,42	0 7 1 36,27 0,55	0 8 7 -0,02 0,07
0 0 9 -0,13 0,1	0 2 7 -0,05 0,05	0 4 2 917,7 12,9	0 5 8 -0,01 0,07	0 7 2 -0,01 0,02	0 8 8 3,2 0,13
0 0 10 1,64 0,15	0 2 8 16,17 0,42	0 4 3 0 0,01	0 5 9 2,57 0,13	0 7 3 19,62 0,32	0 8 9 0,04 0,06
0 1 0 0,02 0,01	0 2 9 -0,07 0,07	0 4 4 38,66 0,49	0 5 10 -0,05 0,05	0 7 4 0 0,02	0 9 0 0,04 0,04
0 1 1 261,7 3,41	0 2 10 5,59 0,21	0 4 5 0,04 0,03	0 6 0 4,11 0,11	0 7 5 30,68 0,47	0 9 1 8,7 0,17
0 1 2 0,01 0,01	0 3 0 0,01 0,01	0 4 6 9,81 0,26	0 6 1 0,03 0,02	0 7 6 0,01 0,04	0 9 2 0 0,03
0 1 3 14,24 0,2	0 3 1 24,79 0,38	0 4 7 -0,02 0,05	0 6 2 91,44 1,3	0 7 7 15,08 0,37	0 9 3 33,21 0,56
0 1 4 0,03 0,02	0 3 2 0 0,01	0 4 8 3,84 0,14	0 6 3 0,02 0,02	0 7 8 0,02 0,07	0 9 4 -0,01 0,03
0 1 5 38,4 0,59	0 3 3 167,8 1,93	0 4 9 0 0,07	0 6 4 7,84 0,13	0 7 9 2,87 0,12	0 9 5 13,61 0,2
0 1 6 0 0,04	0 3 4 0 0,02	0 4 10 3,53 0,14	0 6 5 0,03 0,03	0 7 10 0,01 0,05	0 9 6 -0,04 0,08
0 1 7 10,88 0,21	0 3 5 25,77 0,47	0 5 0 0 0,02	0 6 6 29,02 0,46	0 8 0 3,62 0,12	0 9 7 10,61 0,27
0 1 8 0,01 0,07	0 3 6 0,04 0,04	0 5 1 292 4,1	0 6 7 0,08 0,07	0 8 1 0,02 0,02	0 9 8 0,01 0,06
0 1 9 7,38 0,23	0 3 7 15,01 0,26	0 5 2 0 0,02	0 6 8 3,42 0,15	0 8 2 6,89 0,13	0 9 9 1,89 0,09
0 1 10 -0,01 0,06	0 3 8 0,01 0,08	0 5 3 9,78 0,14	0 6 9 0,02 0,07	0 8 3 -0,02 0,03	
0 2 0 166,8 3,02	0 3 9 4,4 0,2	0 5 4 0,04 0,02	0 6 10 8,95 0,26	0 8 4 311,2 4,2	

glide plan $\perp a$?

Okl with
 $l = 2n$

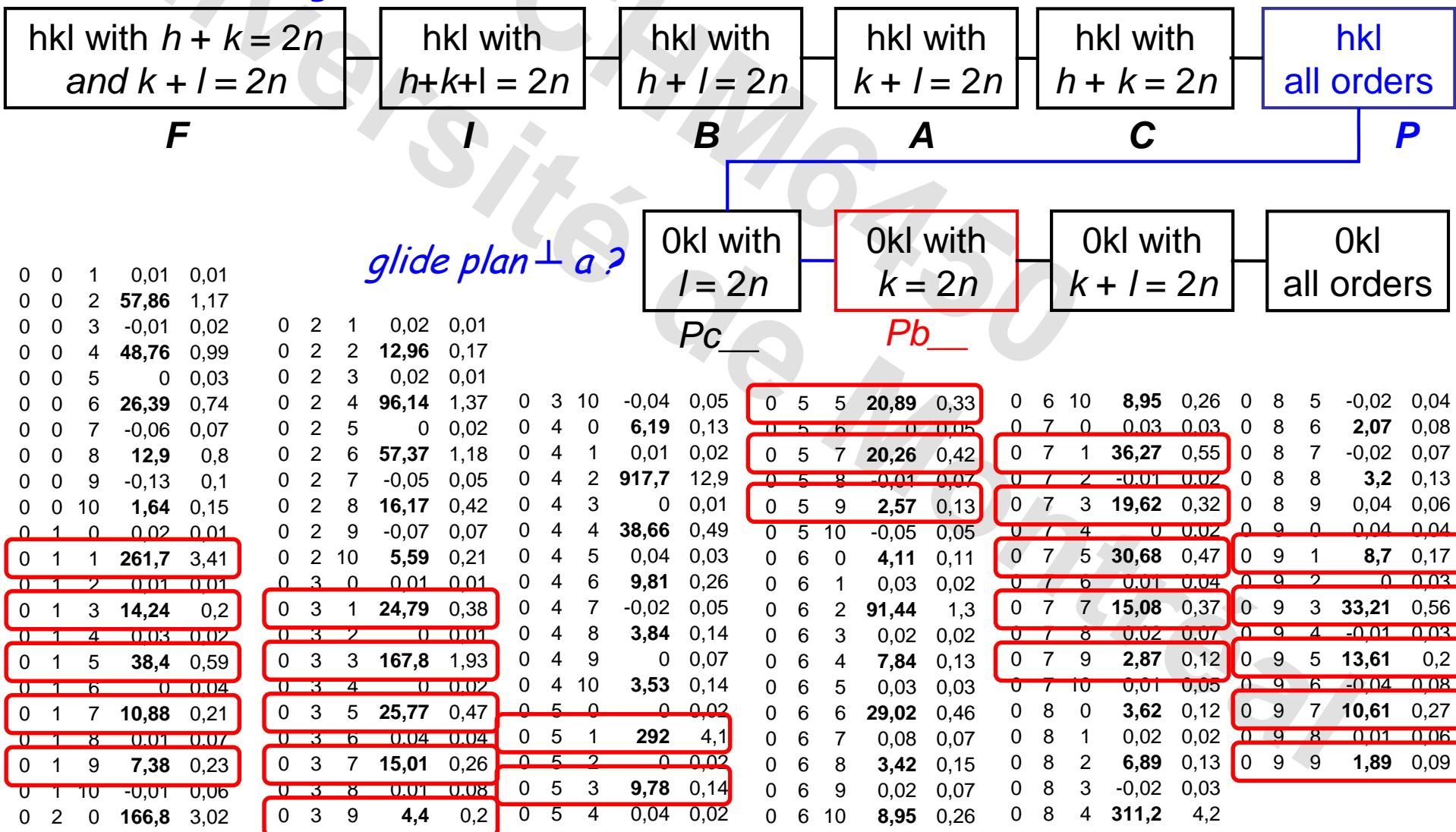
Pc

Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

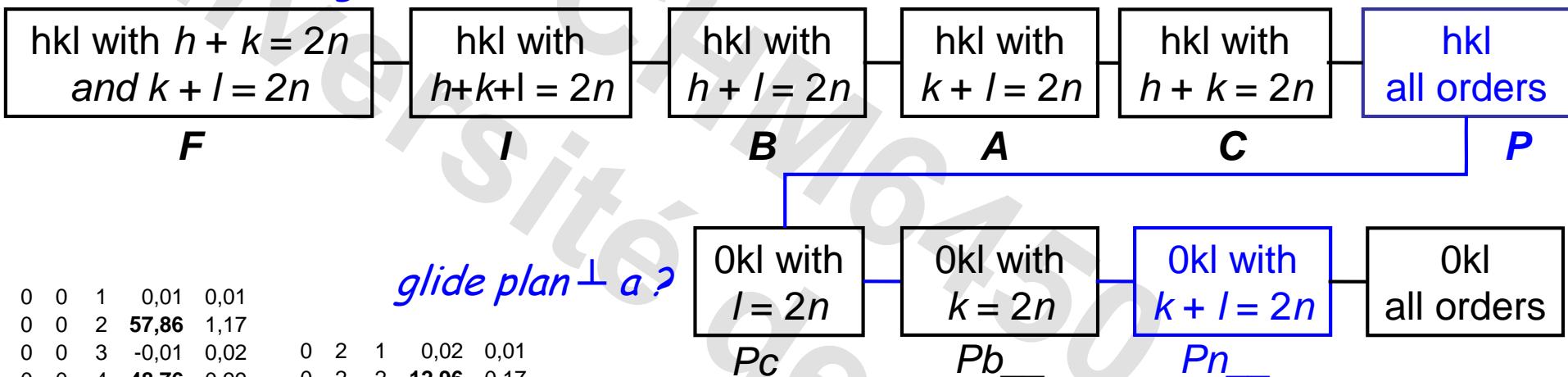


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Test for centering



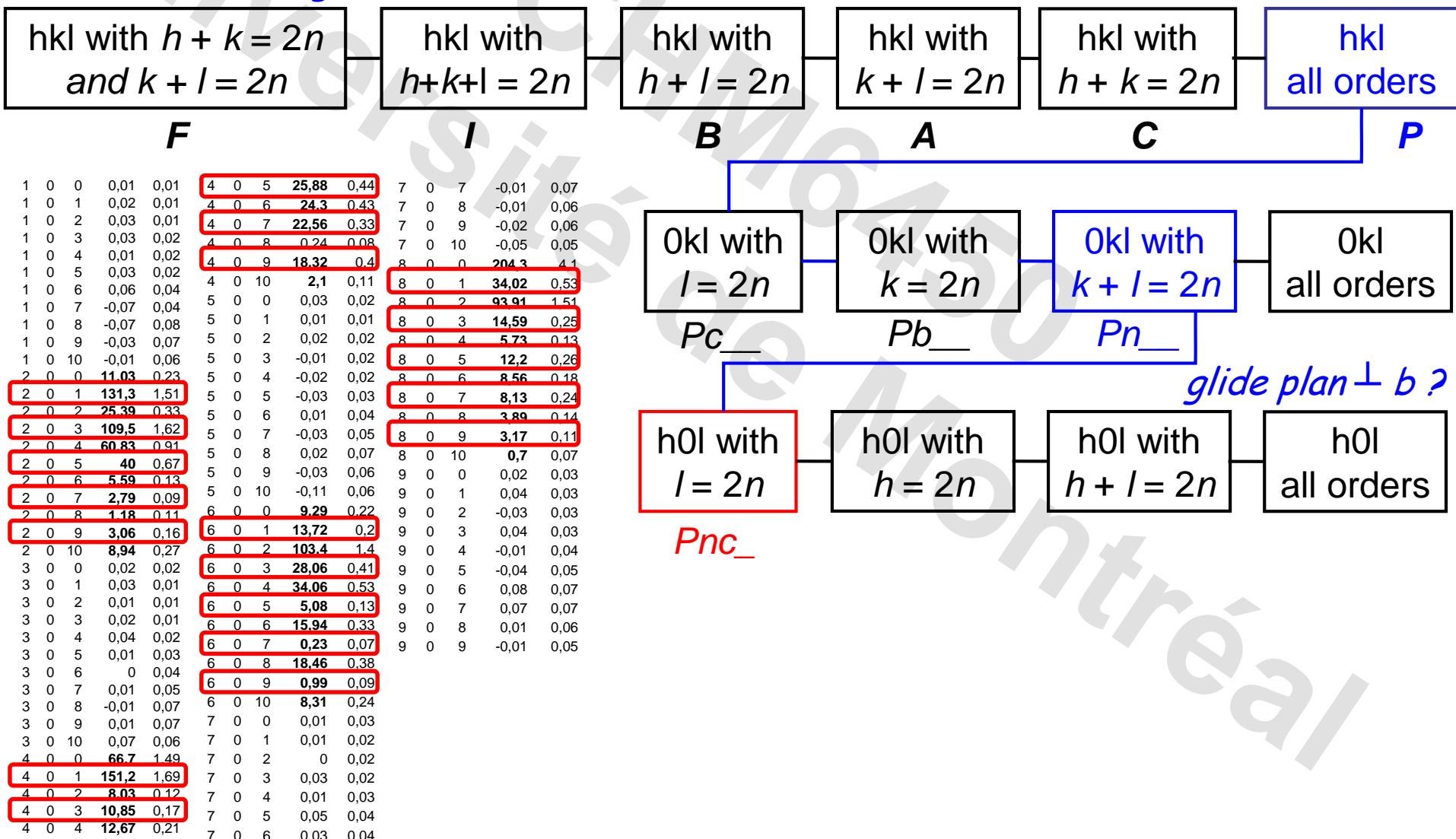
glide plan \perp a ?

Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

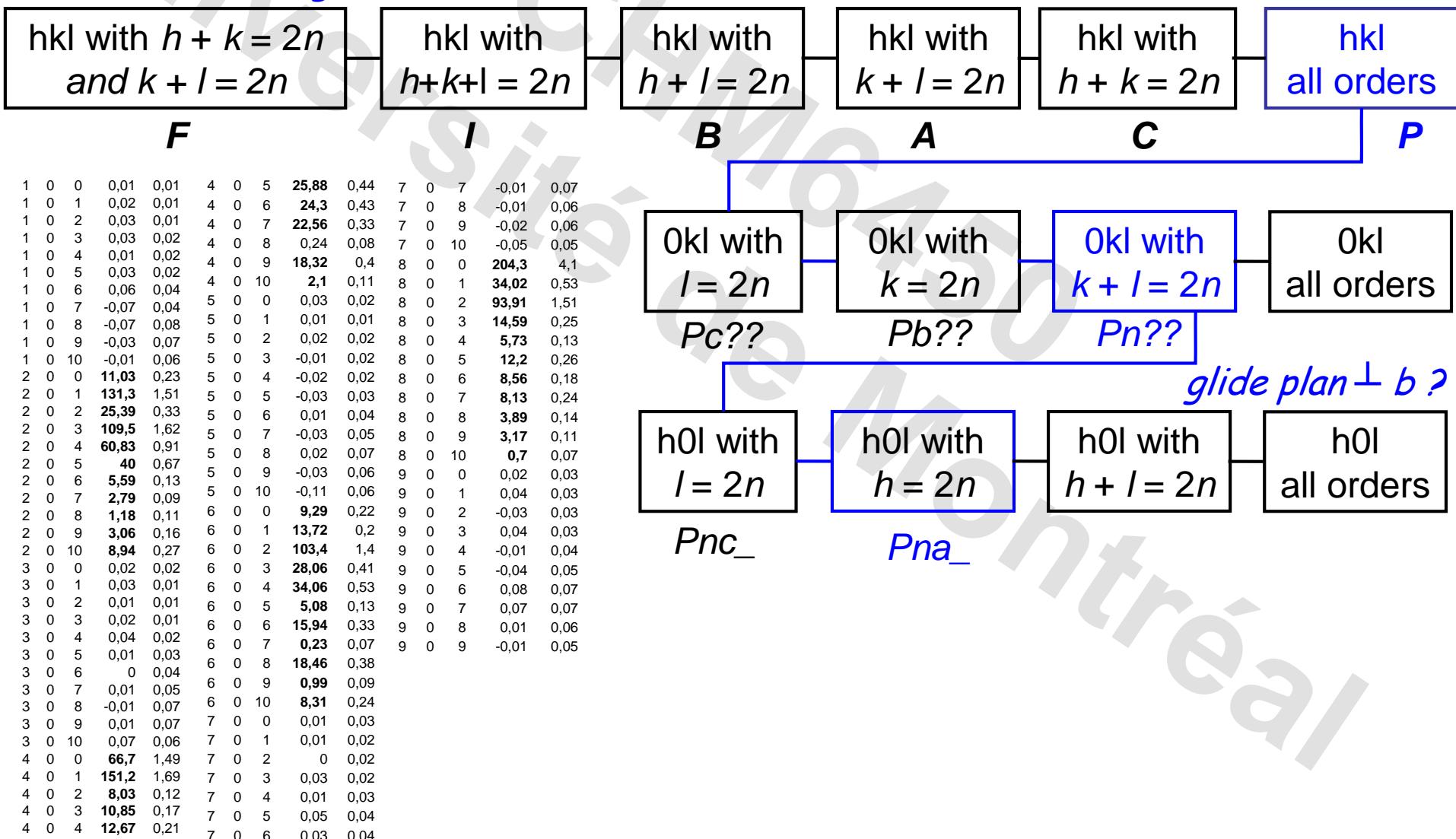


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Test for centering

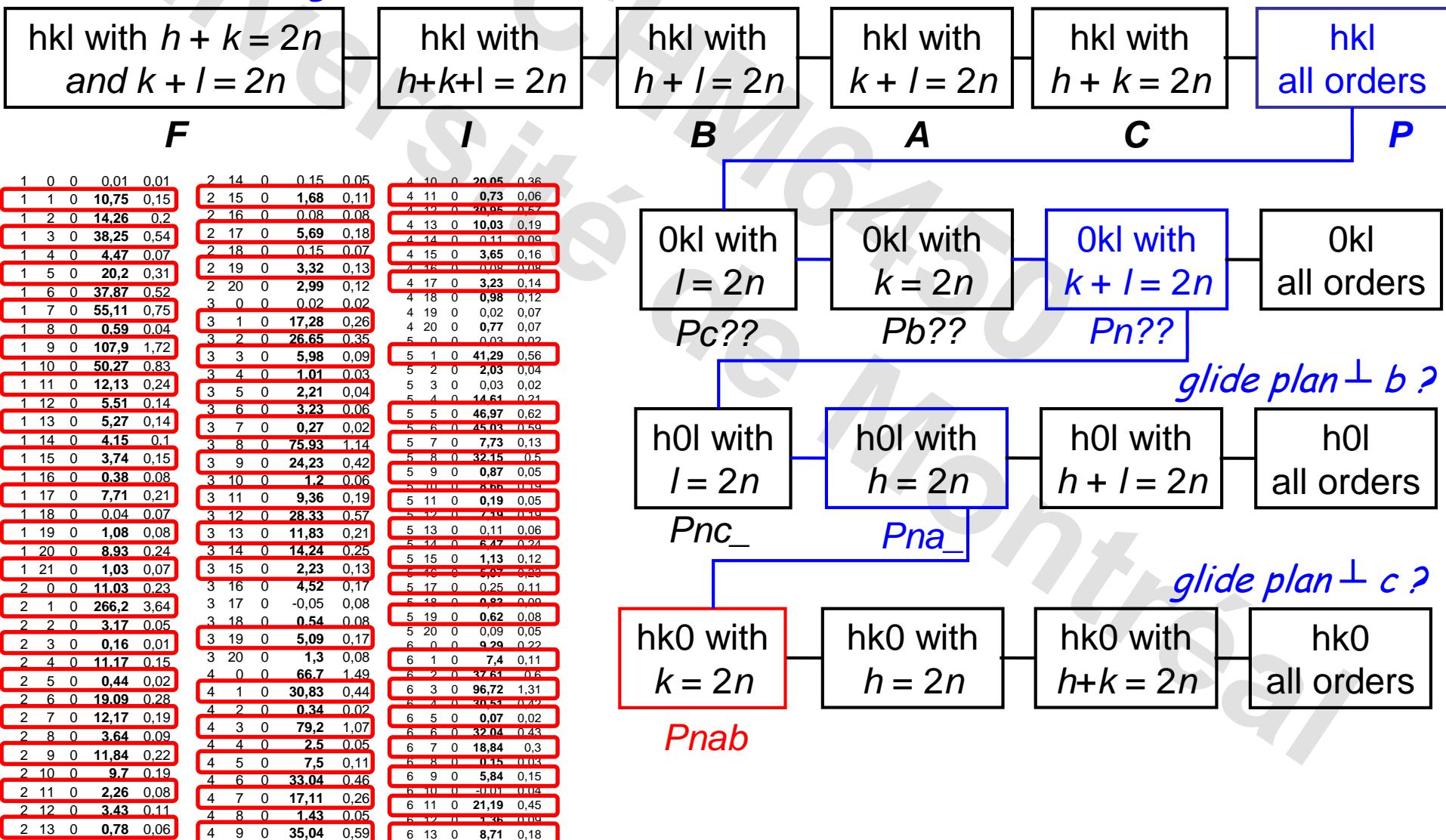


Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

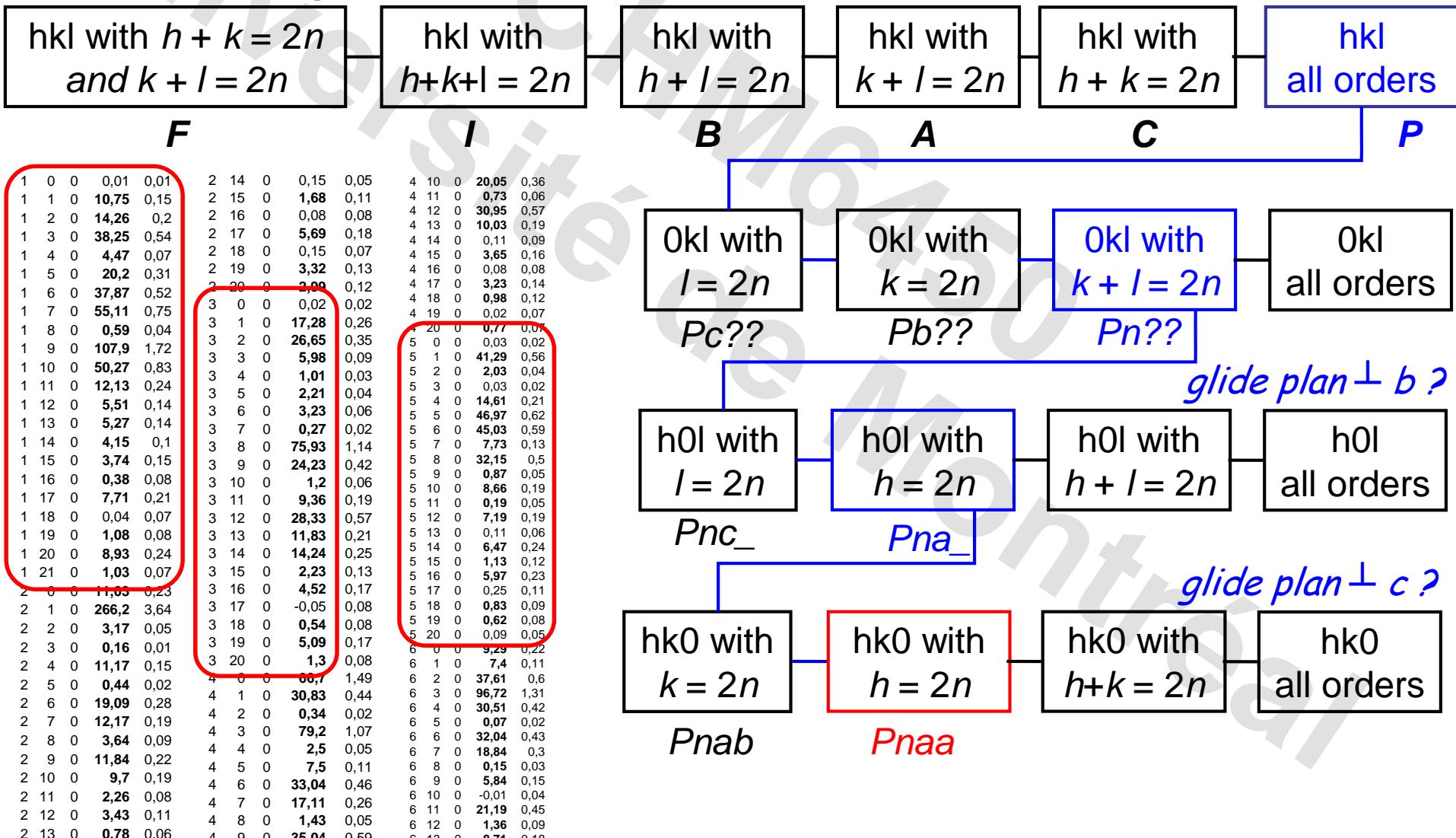


Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

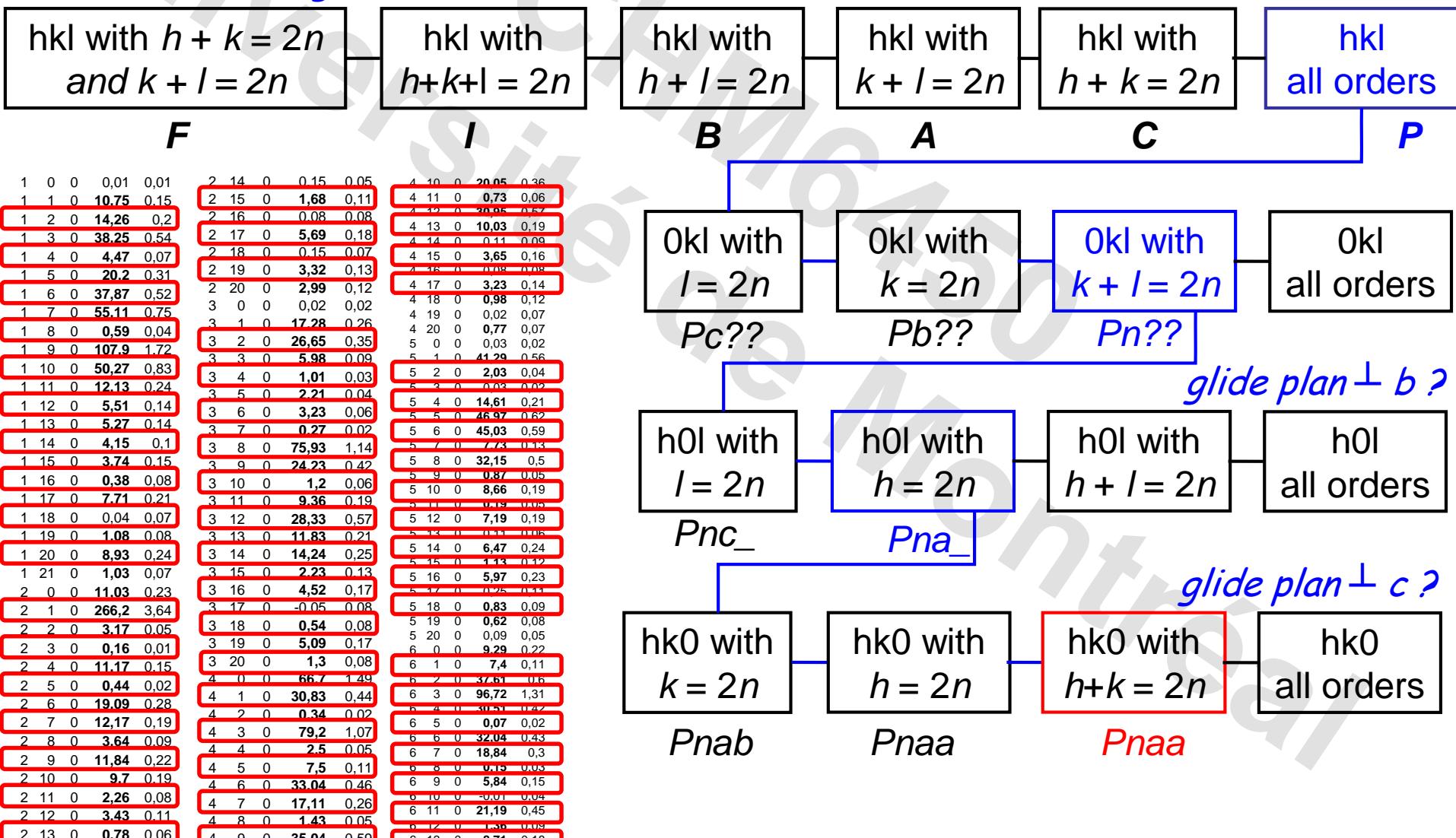


Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

Test for centering

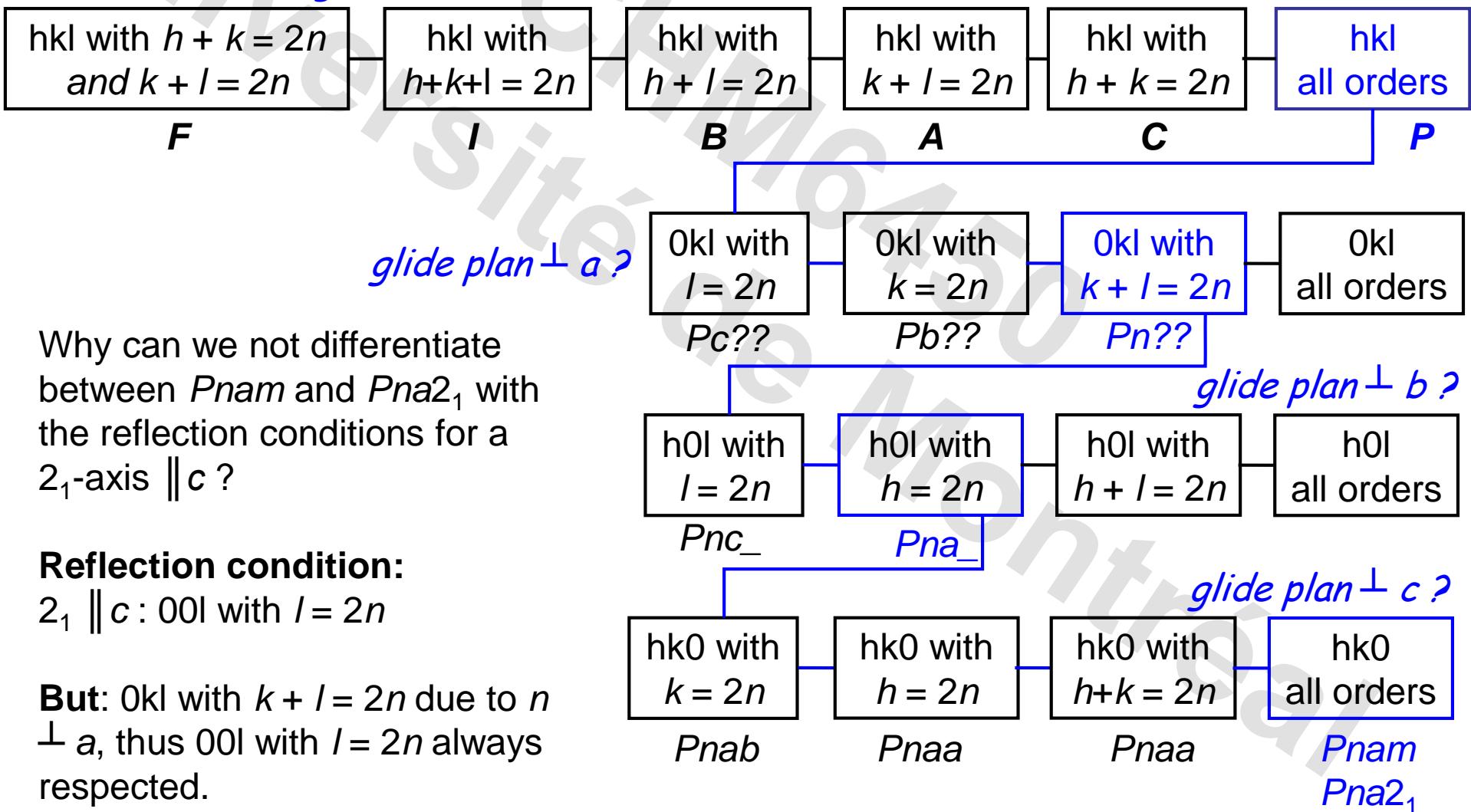


Space group determination by hand

Example Paul32:

$a = 18.4398(3) \text{ \AA}$, $b = 17.4848(3) \text{ \AA}$, $c = 8.9597(2) \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

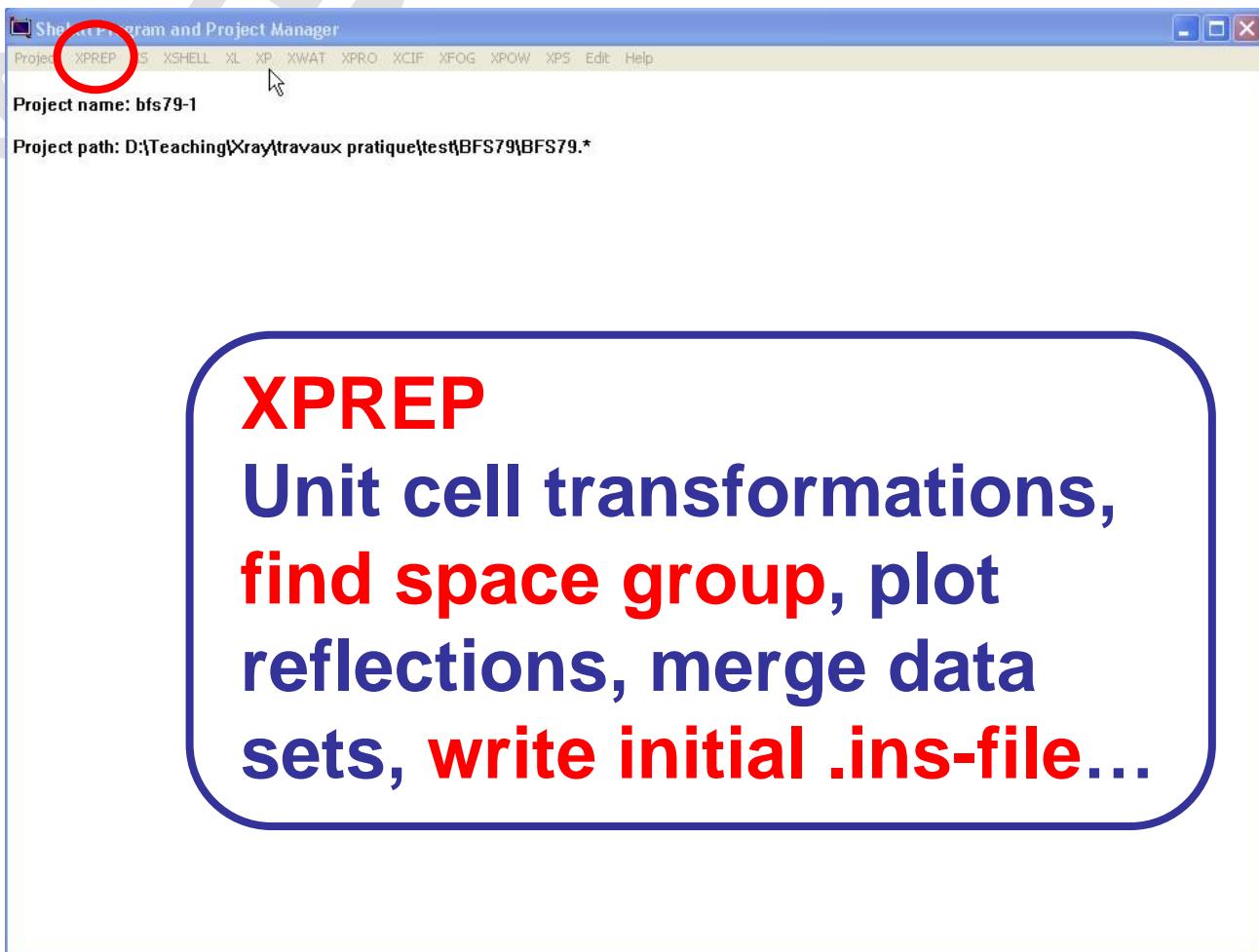
Test for centering



Systematic absences in XPREP

- Determination which screw axes and glide planes are present.
- Determination which space groups contain these symmetry elements.

- By hand
- With XPREP



Space group determination with XPREP

Determination of the possible Bravais lattice: Often already done by a previous program

Reduced (Niggli) cell

Primitive unit cell with $a < b < c$ and all angles either $>/= 90^\circ$ or $</= 90^\circ$. For any crystal there is only **one** cell which fulfills these conditions. The reduced cell is useful to compare unit cells, since it is independent from user choices.

The screenshot shows the BIGXPREP software interface version 5.1. The window title is "BIGXPREP - [VS.1 Copyright (c) 1997 Bruker AXS]". The current dataset is "schapling.hkl". The wavelength is set to 0.71073 Å and the chirality to ?.

Key parameters displayed:

- Original cell: $a = 12.962 \text{ \AA}$, $b = 12.721 \text{ \AA}$, $c = 15.107 \text{ \AA}$, $\alpha = 90.00^\circ$, $\beta = 97.67^\circ$, $\gamma = 90.00^\circ$, Volume = 2468.6 \AA^3 , Lattice = P.
- Current cells: $a = 12.962 \text{ \AA}$, $b = 12.721 \text{ \AA}$, $c = 15.107 \text{ \AA}$, $\alpha = 90.00^\circ$, $\beta = 97.67^\circ$, $\gamma = 90.00^\circ$, Volume = 2468.6 \AA^3 .
- Matrix: $\begin{pmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$.

Available options:

- (D) Read, modify or merge DATASETS
- (P) Contour PATTERSON sections
- (H) Search for HIGHER metric symmetry
- (S) Determine or input SPACE GROUP
- (A) Apply ABSORPTION corrections
- (L) Reset LATTICE type of original cell
- (C) Define unit-cell CONTENTS
- (F) Set up shelxlt FILES
- (R) RECIPROCAL space displays
- (U) UNIT-CELL transformations
- (T) Change TOLERANCES
- (Q) QUIT program

Select option [H]:

Determination of reduced (Niggli) cell

Transformation from original cell (MHLF-matrix):
0.0000 -1.0000 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 1.0000

Unitcell: $a = 12.721 \text{ \AA}$, $b = 12.962 \text{ \AA}$, $c = 15.107 \text{ \AA}$, $\alpha = 97.67^\circ$, $\beta = 90.00^\circ$, $\gamma = 90.00^\circ$

Niggli form: $a.a = 161.82^\circ$, $b.b = 168.01^\circ$, $c.c = 228.21^\circ$, $b.c = -26.15^\circ$, $a.c = 0.00^\circ$, $a.b = 0.00^\circ$

Search for higher METRIC symmetry

Option A: FOM = 0.000 deg., MONOCLINIC, P-lattice, R(int) = 0.026 [10179]
Cell: $a = 12.962 \text{ \AA}$, $b = 12.721 \text{ \AA}$, $c = 15.107 \text{ \AA}$, $\alpha = 90.00^\circ$, $\beta = 97.67^\circ$, $\gamma = 90.00^\circ$, Volume = 2468.55 \AA^3
Matrix: $\begin{pmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$

Option B retains original cell

Select option [A]:

Unit cells the
program finds
acceptable.

Space group determination with XPREP

Confirmation of the Bravais lattice:

Crystal system

Centering

Unit cell employed by the program(s)

Original unit cell

```
BIGXPREP - [V5.1 Copyright (c) 1997 Bruker AXS]

Current dataset: schap1mg.hkl Wavelength: 0.71073 Chiral: ?
Original cell: 12.962 12.721 15.107 90.00 97.67 90.00 Vol 2468.6
Esds: 0.000 0.000 0.000 0.00 0.00 0.00 Lattice: P

Current cell: 12.962 12.721 15.107 90.00 97.67 90.00 Vol 2468.6

Matrix: 1.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 1.0000

Crystal system: Monoclinic Lattice: P

[S] Determine SPACE GROUP
[C] Must be CHIRAL (sample is optically active)
[N] NOT NECESSARILY chiral (eg. may be racemate)
[I] INPUT known space group
[E] EXIT to main menu or [Q] QUIT program

Select option [S]: [A] Triclinic, [M] Monoclinic, [O] Orthorhombic, [T] Tetragonal,
[H] Trigonal/Hexagonal, [C] Cubic or [E] EXIT

Select option [M]: [P] P [A] A [B] B [C] C [I] I [F] F [O] Obv [R] Rev [All]

Lattice exceptions: P Å B C I F Obv Rev All
N (total) = 0 7643 7646 7629 7643 11459 10187 10175 15264
N (int>3sigma) = 0 5858 5714 5868 5885 8720 7755 7760 11691
Mean intensity = 0.0 22.2 21.4 21.5 20.7 21.7 22.3 21.9 22.0
Mean int/sigma = 0.0 20.2 19.8 19.9 19.8 20.0 20.2 20.2 20.1

Lattice type [P, A, B, C, I, F, O(obv.), R(rev. rhomb. on hex. axes)]

Select option [P]:
```

Space group determination with XPREP

Reflections which **violate** the reflection condition. For **A** centering: $k + l = 2n$

Of the reflections above, those with **non-zero intensity**

Lattice exceptions: P	A	B	C	I	F	Obv	Rev	All
N (total) =	0	7643	7646	7629	7643	11459	10187	10175
N (int>3sigma) =	0	5858	5714	5868	5885	8720	7755	7760
Mean intensity =	0.0	22.2	21.4	21.5	20.7	21.7	22.3	21.9
Mean int/sigma =	0.0	20.2	19.8	19.9	19.8	20.0	20.2	20.1

There are no reflection conditions for a primitive cell

Average intensity and signal noise ratio for these reflections

Compare to the values for all reflections

A
If we would have
A centering:
7643
858
1.3
0.2

Space group determination with XPREP

Determination of the space group using systematic absences

```
Systematic absence exceptions:  
  
      -21-    -a-    -c-    -n-  
  
N       16     369     364     371  
N I>3s   0      168     169      11  
<I>     0.1    32.0    32.5     0.1  
<I/s>   0.8    18.3    18.7     0.8
```

```
[XPS] may be racemate)  
IT program  
  
[O] Orthorhombic, [T] Tetragonal,  
c or [E] EXIT  
  
B      C      I      F      Obv      Rev      All  
646    7629    7643    11459    10187    10175    15264  
714    5868    5885    8720     7755    7760     11691  
1.4    21.5    20.7    21.7     22.3    21.9     22.0  
9.8    19.9    19.8    20.0     20.2    20.2     20.1  
  
[obv.), R(rev. rhomb. on hex. axes)]
```

Systematic
absences

```
Mean |E*E-1| = 0.980 [expected .968 centrosym and .736 non-centrosym]  
  
Systematic absence exceptions:  
  
      -21-    -a-    -c-    -n-  
N       16     369     364     371  
N I>3s   0      168     169      11  
<I>     0.1    32.0    32.5     0.1  
<I/s>   0.8    18.3    18.7     0.8  
  
Option Space Group No. Type Axes CSD R(int) N(eq) Syst. Abs. CFOM  
[A] P2(1)/n      # 14 centro 1 19410 0.026 10179 0.8 / 18.3 0.59  
  
Select option [A]:
```

Again, compare
with the values for
all reflections

Space group determination with XPREP

Determination of the space group using systematic absences

```
BIGXPREP - [V5.1 Copyright (c) 1997 Bruker AXS]
[N] NOT NECESSARILY chiral (eg. may be racemate)
[I] INPUT known space group
[E] EXIT to main menu or [Q] QUIT program

Select option [S]:
```

```
[A] Triclinic, [M] Monoclinic, [O] Orthorhombic, [T] Tetragonal,
[H] Trigonal/Hexagonal, [C] Cubic or [E] EXIT
```

```
Select option [M]:
```

Lattice exceptions:	P	A	B	C	I	F	Obv	Rev	All
N (total) =	0	7643	7646	7629	7643	11459	10187	10175	15264
N (int>3sigma) =	0	5858	5714	5868	5885	8720	7755	7760	11691
Mean intensity =	0.0	22.2	21.4	21.5	20.7	21.7	22.3	21.9	22.0
Mean int/sigma =	0.0	20.2	19.8	19.9	19.8	20.0	20.2	20.2	20.1

```
Lattice type [P, A, B, C, I, F, O(obv.), R(rev. rhomb. on hex. axes)]
```

```
Select option [P]:
```

```
Mean |E*E-1| = 0.980 [expected .968 centrosym and .736 non-centrosym]
```

```
Systematic absence exceptions:
```

```
-21- -a- -c- -n-
```

N	16	369	364	371
N I>3s	0	168	169	11
<I>	0.1	32.0	32.5	0.1
<I/s>	0.8	18.3	18.7	0.8

```
Option Space Group No. Type Axes CSD R(int) N(eq) Syst. Abs. CFOM
```

```
[A] P2(1)/n      # 14 centro   1 19410  0.026 10179  0.8 / 18.3  0.59
```

```
Select option [A]:
```

E-statistics

Statistical evidence for an inversion center

The theoretical distribution of intensities is different for centrosymmetric and non-centrosymmetric structures.

Since the intensities depend on the reflection angle θ , we use **normalized structure factors E**.

$$E_{hkl}^2 = \frac{I_{hkl}}{\langle I_{hkl} \rangle_\theta}$$

Centrosymmetric:

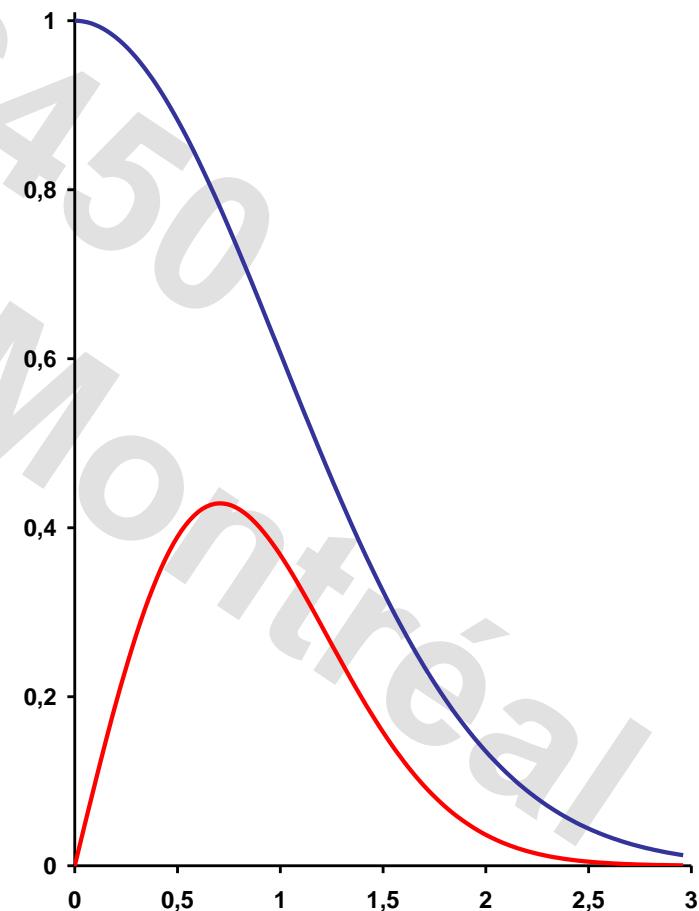
Greater probability for strong or weak reflections

$$P(E) = e^{-\frac{1}{2}|E|^2}$$

Non-centrosymmetric:

Distribution around an average value

$$P(E) = |E|e^{-|E|^2}$$



E statistics

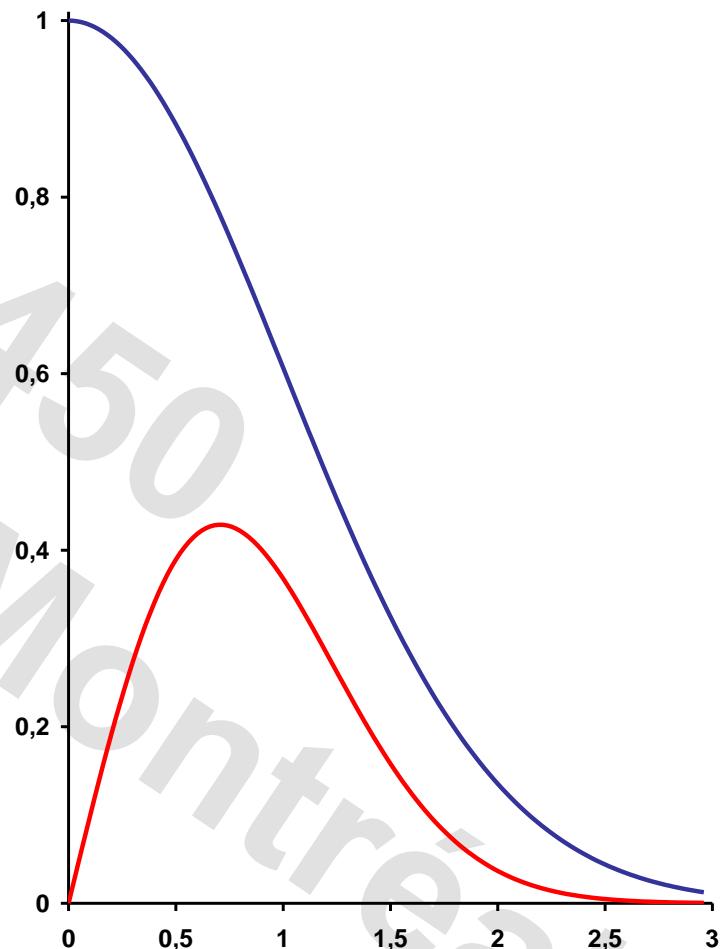
centrosym.

$$P(E) = e^{-\frac{1}{2}|E|^2}$$

non-centrosym.

$$P(E) = |E|e^{-|E|^2}$$

$ E < 1$	63.1%	$ E > 1$	61.4%
$ E > 1$	31.7%	$ E > 2$	36.8%
$ E > 2$	4.6%	$ E > 3$	1.8%
$ E > 3$	0.3%	$\langle E ^2 \rangle$	0.01%
$\langle E ^2 \rangle$	1.000	$\langle E^2 - 1 \rangle$	1.000
$\langle E^2 - 1 \rangle$	0.968	$\langle E \rangle$	0.736
$\langle E \rangle$	0.798		0.886



E statistics

centrosym. non-centrosym.

$\langle E^2 - 1 \rangle$	0.968	0.736
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- Real structures do not have a uniform distribution of atoms and the values might differ from the theoretical ones.
- **0.74-0.97:** A non-centrosymmetric structure easily can have tendencies towards centrosymmetry, without fulfilling it completely. A value of 0.85 indicates thus more likely a non-centrosymmetric than a centrosymmetric space group.
- **»0.74:** A value notably smaller than 0.74 might indicate twinning.
- **»0.97:** A value much higher than 0.97 might indicate “hypersymmetry”: The asymmetric unit of a centrosymmetric structure is nearly centrosymmetric itself.

Space group determination with XPREP

Determination of the space group using systematic absences

Option	Space Group	No.	Type	Axes	CSD	R(int)	N(eq)	Syst. Abs.	CFOM
[A]	P2(1)/n	# 14	centro	1	19410	0.026	10179	0.8 / 18.3	0.59

Select option [A] :

If several space groups seem possible, you have to try all of them!

We will deal with problems here later.

Acceptable space groups

[H] Trigonal/Hexagonal, [C] Cubic or [E] EXIT
Select option [M] :

Lattice exceptions: P A B C I F Obv Rev All
N (total) = 0 7643 7646 7629 7643 11459 10187 10175 15264
N (int>3sigma) = 0 5858 5714 5868 5885 8720 7755 7760 11691
Mean intensity = 0.0 22.2 21.4 21.5 20.7 21.7 22.3 21.9 22.0
Mean int/sigma = 0.0 20.2 19.8 19.9 19.8 20.0 20.2 20.2 20.1

Lattice type [P, A, B, C, I, F, O(obv.), R(rev. rhomb. on hex. axes)]

Select option [P] :

Mean |E*E-1| = 0.980 [expected .968 centrosym and .736 non-centrosym]

Systematic absence exceptions:

-21- -a- -c- -n-

N 16 369 364 371
N I>3s 0 168 169 11
<I> 0.1 32.0 32.5 0.1
<I/s> 0.8 18.3 18.7 0.8

Option	Space Group	No.	Type	Axes	CSD	R(int)	N(eq)	Syst. Abs.	CFOM
[A]	P2(1)/n	# 14	centro	1	19410	0.026	10179	0.8 / 18.3	0.59

Select option [A] :

Ambiguities in space group determination

36770 reflections read from file paul32.hkl Mean I/sig(I) = 10.55

Lattice exceptions	P	A	B	C	I	F	Obv	Rev	All
Total no. reflections	0	18455	18302	18397	18426	27577	24453	24449	36770
Total with I>3sig(I)	0	14415	14946	14937	14677	22149	19920	19917	30028
Mean I/sig(I) (all)	0.0	10.1	10.5	10.5	10.3	10.3	10.5	10.5	10.5
Mean I/sig(I) I>3sig(I)	0.0	12.6	12.6	12.7	12.7	12.6	12.6	12.6	12.6

Suggested lattice type is P

R(int) values for merging under all Laue symmetries :

Laue class	R(int)	N(obs)	N(ind)	R1	<n>
-1	0.039	36552	10109	0.045	3.616
2/m	0.043	36746	5469	0.046	6.719
mmm	0.044	36764	3022	0.046	12.165
4/m	0.563	36760	2924	0.582	12.572
4/mmm	0.566	36768	1633	0.577	22.516
-3 (rhomb)	0.700	36645	5389	0.731	6.800
-3m (rhomb)	0.803	36735	3112	0.823	11.804
-3 (hex)	0.720	36689	5228	0.749	7.018
-3m1 (hex)	0.804	36748	2911	0.825	12.624
-31m (hex)	0.806	36746	2855	0.825	12.871
6/m	0.729	36759	2835	0.745	12.966
6/mmm	0.809	36767	1632	0.821	22.529
m-3	0.710	36767	1630	0.720	22.556
m-3m	0.811	36769	951	0.818	38.664
2/m 1 1	0.042	36748	5462	0.045	6.728
1 1 2/m	0.043	36739	5628	0.046	6.528

Highest diffraction symmetry with reasonable value of R(int) is mmm

Highest diffraction symmetry compatible with the cell metrics is mmm

Group	Cond.	Op.	All	Odd	Cut1	Cut2	Cut3	$\langle I/sI \rangle$	Op.	No.
h00	$h=2n+1$		92	47	0	0	0	0.0	21..	1
0k0	$k=2n+1$		96	48	0	0	0	0.0	.21..	2
001	$l=2n+1$		39	23	0	0	0	0.0	.21..	3
0kl	$k=2n+1$	b..	1395	718	269	238	179	6.0		4
0kl	$l=2n+1$	c..		702	269	238	179	6.2		5
0kl	$k+l=2n+1$		710	0	0	0	0	0.0	n..	6
h01	$h=2n+1$		1441	734	0	0	0	0.0	.a..	7
h01	$l=2n+1$.c..		736	279	239	192	6.0		8
h01	$h+l=2n+1$.n..		708	279	239	192	6.3		9
hk0	$h=2n+1$..a	2663	1321	707	487	296	6.7		10
hk0	$k=2n+1$..b		1327	650	479	290	6.5		11
hk0	$h+k=2n+1$..n		1330	665	486	280	6.4		12
hkl	$k+l=2n+1$	A..	18432		8906	5822	3328	5.7		13
hkl	$h+l=2n+1$.B..	18279		9231	5983	3491	6.1		14
hkl	$h+k=2n+1$..C	18397		9395	6215	3385	6.0		15
hkl	$h+k+l=2n+1$	I	18403		9222	6264	3465	6.0		16
hkl	not all odd/even	F	27554		13766	9010	5102	5.9		17
h00	$h=4n+1$	41..		73	17	17	5	3.1		18
0k0	$k=4n+1$.41..		74	26	16	9	3.9		19
001	$l=4n+1$..41		33	8	8	8	5.4		20
0kl	$k+l=4n+1$	d..	1045		259	215	149	3.6		21
h01	$h+l=4n+1$.d..	1074		403	356	253	5.8		22
hk0	$h+k=4n+1$..d	1976		1011	722	422	6.5		23

Candidate space groups (NS = non-standard setting) :

H-M symbol	No.	Centric	Laue class	M	R(int)	N(obs)	CSD	ICSD	CFOM
P n a 21	33	no	mmm	4	0.044	36764	3013	313	4.399
P n a m NS	62	yes	mmm	8	0.044	36764	3005	2296	4.399

The program cannot distinguish between these two space groups. A proposition will be made (often) from E-statistics and general frequency of occurrence.